Mandatory pension savings, private savings, homeownership, and financial stability

By
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Abstract. This paper contributes to the discussion of effects of mandatory pension savings and house price risk on aggregate household savings, homeownership, and risks in lending to homeowners. The analysis is theoretical and based on the life-cycle hypothesis. It is shown that mandatory pension savings based on defined benefits will increase risk in lending to homeowners. Households that remain homeowners will increase their personal savings while those that prefer renting will decrease their savings as renters take on less risk from house price volatility than homeowners. The relative size of the two effects on savings depends on households' preferences over homeownership and renting. The assets of the mandatory pension funds in Iceland are among the highest in the world. This country also scores very high in homeownership with around 80% of households living in own homes. For these reasons data on the Icelandic pension system and on homeownership in this country provide a convenient background for discussion of the theoretical issues.

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1. Introduction

Saving for retirement is probably the most important motive for savings in modern economies. To be able to afford decent living during retirement households must save a large part of the income they earn during their working life. Pension savings of between 10% and 20% of the wage income is considered appropriate in many countries. In countries where there are specialized pension funds they have accumulated vast sums of money. By the end of 2004 pension funds in Iceland had net assets slightly in excess of 110% of GDP which brought the country to the top of the list of OECD countries. Switzerland was almost on level with Iceland and Netherlands were also above 100%. US was in the fourth place with net assets amounting to 95% of GDP, Australia was in the fifth place with net assets slightly above 70% and UK was in the sixth place with net assets worth 65% of GDP. By the end of 2009 Iceland had dropped to second place with net assets amounting to 118% of GDP. Netherlands were at the top with net assets amounting to 130% of GDP and Switzerland was in the third place with assets amounting to 101% of GDP.[1] These large assets of the pension funds in Iceland have frequently led commentators discussing the longer term fiscal pressures in Iceland to point out correctly that "fully-funded occupational and public-employee pension funds limit the effects of population ageing." (OECD 2008, p. 15)

There exists now a large literature discussing effects of mandatory pension savings on total savings. Most of this literature uses the life-cycle hypothesis of Franco Modigliani (see Ando and Modigliani, 1963) to explain the determinants of savings. This theory predicts that if mandatory pension savings are introduced in a country where pension rights are very limited and most households' pensions has to come from personal savings, they will reduce other personal savings. Many writers report evidence that the introduction of pension funds while reducing households' other savings actually increases total savings. This evidence is though not conclusive. (See e.g. Már Guðmundsson, 2001 and Kohl, R., P. O'Brien, 1998, Schmidt-Hebbel, 1999 and Bloom e al., 2006.)

The argument for making pension savings mandatory is that voluntary savings by households for retirement are inadequate, possibly because they are not foresighted enough, especially when they are young and old age seems very far off, or because they know that they will be provided for to some extent independently of their private savings, i.e. there is some moral hazard involved. (See Modigliani and Muralidhar, 2004, p. 1, and references therein)

There are different types of pension funds. Some writers prefer a system where the right to pension is defined on the basis of households' contributions and where contributions that are not paid out as a pension will be left for inheritance. In such cases the pension rights of individual households are very close to being a personal property and in some cases these entitlements can be used as collaterals. Modigliani and Muralidhat (2004) argue for a system where benefits are defined. There are greater financial risks associated with pension funds based on defined benefits but these funds provide income insurance for the entire retirement for their members and if the funds are cleverly managed they may also provide some income insurance between generations as pointed out in Modigliani and Muralidhar (2004).

The main contribution of this paper is a theoretical discussion of the effects that mandatory pension savings and house price risk have on aggregate household savings, homeownership and risks in lending to homeowners. It is shown that the insurance element in the pension system with defined benefits will result in less savings than where the pension system is based on contributions or where there is no pension system and each household saves for its retirement. It is shown that mandatory pension savings based on defined benefits will increase risk in lending to homeowners. Households

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[1] See OECD (2006), p. 71 and OECD (2011b). The Icelandic population is relatively young and the assets of the pension funds in the country are expected to increase during the next ten to twenty years.

that remain homeowners will therefore increase their personal savings while those that prefer renting will decrease their savings as the price risk that renters face is much smaller than the price risk faced by homeowners. The relative size of the two effects on savings depends on the relative attractiveness of homeownership versus renting, public subsidies and the effectiveness of the markets and the legal frameworks. If renting is very unattractive people will continue to prefer homeownership even when it has become much riskier. In such cases mandatory pension funds may decrease risks to public finances by population ageing but at the same time increase risks in lending to households.

Risks in lending to suppliers of rented accommodation are not included in the analysis in this paper. If mandatory pension funds lead to decreases in homeownerships and possibly to decreases in risks from lending to households, the overall lending risk may actually increase if there are greater risks involved in lending to those that supply rented accommodation as indicated in Gerlach (2012).

The volatility of house prices affects homeowners in two ways. On the one hand it brings capital losses or gains to home owning households, but on the other it changes the cost (i.e. capital cost) of accommodation. It is shown that these two effects are negatively correlated and that the former effect is larger. If the market for rented accommodation is effective and the rent is positively correlated with the house price the renter will gain when the house price decreases but loose when the house price increases, contrary to the homeowner.

The organization of the paper is such that Section 2 discusses data on savings and net assets of pension funds in Iceland. It also discusses homeownership and equity of households. Savings, as measured in the national account, as a share of GDP, exhibits a downward trend during recent decades at the same time as net assets of pension funds were increasing as a share of GDP. Estimates of national wealth based on the sum of the value of fixed capital and the international investment position, as a share of GDP, also exhibits a negative trend during this period. It is also shown that households' savings measured by the net increase in household wealth differ significantly from the national account measures of savings. The reason is that asset prices, especially house prices, and the prices of equity, have fluctuated enormously in Iceland during recent years. These measures show large fluctuations in households' wealth but during the last 20 years they show no decline in households' other savings but an increase in households' total savings with pension funds included.

Sections 3-5 provide theoretical discussions of issues that explain how mandatory pension savings influences other household savings and financial risk taking. In Section 3 a simple model where longevity is the only source of risk is presented. It is shown that insurance against this risk provided by pension funds based on defined benefits leads to less savings than if no such insurance exists. Theoretically, this can explain why introduction of mandatory pension schemes based on defined benefits in a country where there is no such scheme, or where pension schemes are based on contributions, can lead to considerable decrease in aggregate saving. In Section 4 additional risk from house prices is introduced. It is shown that households that prefer homeownership will purchase smaller houses and that they will save more when house prices are volatile. When mandatory pension savings are introduced into this framework and it is assumed that the mandatory pension scheme is organized so that there is no reason for the financial institutions to doubt that households will pay back their debt and therefore they will not need any collateral against their lending and if the financial markets are efficient in providing loans to households so that they do not have to deviate from their optimal consumption path because of liquidity constraints, mandatory pension savings does not affect homeownership or the type of accommodation that people choose. If these assumptions are relaxed mandatory pension scheme will influence households' savings and homeownership. If, as is discussed in Section 5, financial institutions demand collateral against their lending and make households bankrupt if their equity falls below some low level, and bankruptcy is costly for households, homeownership becomes more costly discouraging people from buying their homes and those that do buy their own homes choose smaller accommodations. The risk of

bankruptcy leads homeowners to save more. This increase in savings will be smaller than the contributions to the mandatory pension funds so that personal equity of the homeowners decreases and the risk of bankruptcy increases. The increased risk of bankruptcy will make some household decide to rent rather than buy their own home. This will decrease households' savings as renters will accumulate less precautionary savings than homeowners. The aggregate effect on households' savings is therefore indeterminate. The aggregate financial risk of lending to home owning households is also indeterminate as the number of homeowners declines. Section 6 concludes and discusses some implications for policy.

2. Savings and net assets of pension funds in Iceland

The mandatory pension funds in Iceland are based on defined benefits and they are required to adjust pensions to ensure that their policies are sustainable.[2] The members are entitled to pension for their entire retirement. The level of the pension depends on contributions but the duration of the pension rights does not. The supplementary pension system in Iceland ("séreignar lífeyrissparnaður" in Icelandic) that was introduced in late 1990s to increase personal savings of households is based on contributions. This latter system is relatively small compared to the former system. Those that have paid into the supplementary pension system are entitled to withdraw their pension when they have reached 60.[3]
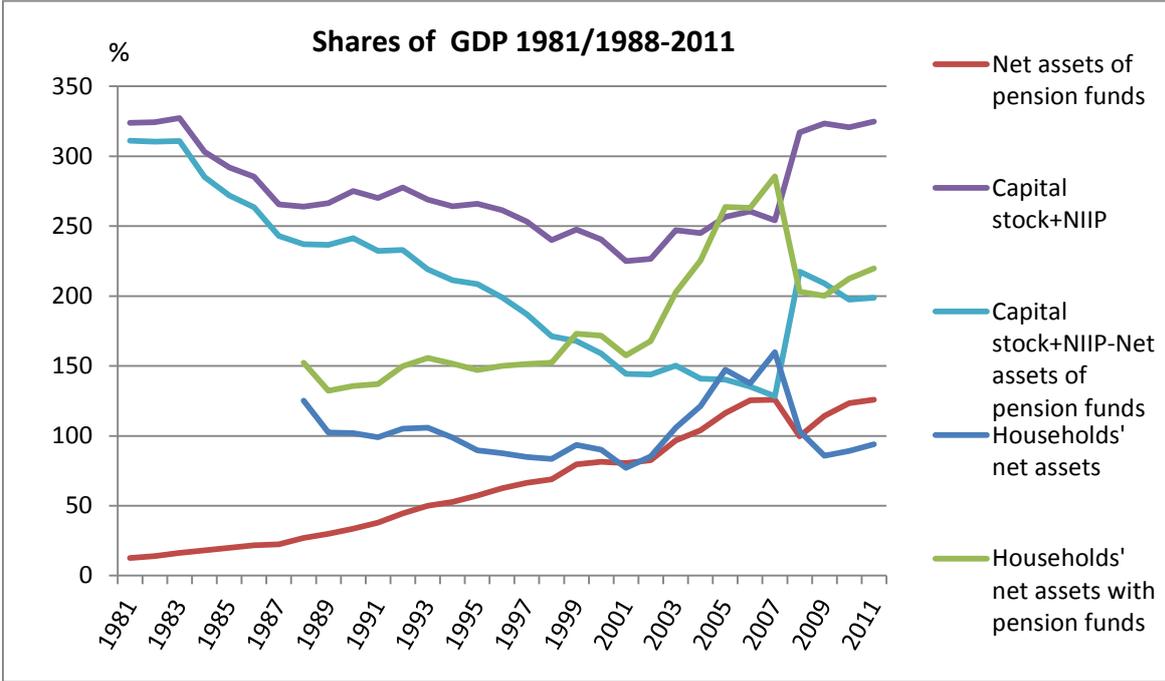
Net assets of the pension funds in Iceland increased from 10.4% of GDP in 1980 to 116% in 2011. This ratio peaked in 2007 when it was 126% but declined to 100% in 2008, immediately after the collapse of the Icelandic banks. Since then the ratio has increased substantially. At the same time the wealth of the Icelandic nation measured by the sum of the value of the fixed capital as estimated by Statistics Iceland and the net international investment position (NIIP) as estimated by the Central Bank of Iceland declined from 327% of GDP in 1983 to 254% in 2007.[4] Since then this ratio has recovered and was 325% by the end of 2011. The pension funds' net assets that were negligible part of the national wealth in the early 1980s amounted to 51% of the wealth of the nation in 2007 but has since declined to 37% of the net national wealth by the end of 2011.

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[2] In law no. 129/1997 (www.althingi.is/lagas/nuna/1997129.html) on mandatory pension insurance and operations of pension funds the fourth paragraph specifies that an individual that has paid into a pension fund for 40 years is entitled to a minimum pension of 56% of the montly wage that was the basis for their contributions to the pension fund.

[3] In the aftermath of the collapse of the banks in 2008 special legislation was passed in Althing (the parliament) allowing households to withdraw from their supplementary pensions savings. This right was limited in time and there was also an upper limit on the amount that each person could withdraw.

[4] This definition of wealth ignores the less tangible wealth like human capital. These types of savings in Iceland are discussed in Þórhildur (1998).

Figure 2.1

Shares of GDP 1981/1988-2011

Legend:
- Net assets of pension funds
- Capital stock+NIIP
- Capital stock+NIIP-Net assets of pension funds
- Households' net assets
- Households' net assets with pension funds

Figure 2.1 shows the above mentioned ratios in greater detail. It also shows the wealth of the nation (Capital stock+NIIP) excluding assets of the pension funds which amounted to 311% of GDP in 1981 but declined steadily and was 128% by the end of 2007. Since then it has increased and was 209% in 2011. In 2008, the year of the collapse of the Icelandic banks, this ratio increased from 128% by the end of 2007 to 217% of GDP by the end of 2008.

Figure 2.1 shows the households' net assets or equity as estimated by the Central Bank of Iceland and published in the data set for the macroeconomic model (QMM). These data are available from 1988. The net assets of households, excluding the assets of the pension funds, declined fairly steadily from 125% of GDP in 1988 to 77% in 2001 and 85% in 2002. During the boom the ratio increased and peaked in 2007 when it was estimated 160% of GDP. After the failures of the banks and the decline in asset prices this ratio declined and was 94% in 2011.

The sum of net assets of pension funds and net assets of households as a share of GDP exhibits an increasing trend from 152% in 1988 and 132% in 1989 to 286% in 2007.[5] In 2011 the ratio had declined to 210%. It should be noted that net assets of households, inclusive of the assets of pension funds, exceeded the wealth of the nation, as defined above, during the three year period 2005-2007. This example shows clearly that different methods of estimation may lead to very different estimates of the value of assets and of savings. Methods based on market prices of financial assets often produce very different picture from the one given by national accounting methods. The data behind the figures on the wealth of the nation are based on national accounting methods while the figures on the net assets of pension funds and on the net assets of households include effects of changes in asset prices. Obstfeld (2012) discusses the differences between changes in the net international position of a country on the one hand and the current account on the other. He notes large differences but warns against concluding from this that the current account is not important.

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[5] The tax system treats these assets differently. Payments into into the pension funds are not taxed but the pensions are subjected to income tax. And while the personal wealth of households are subjected to capital-income taxes the capital incomes of pension funds are not taxed. This means that the value of the net assets of pension funds is not directly comparable to households' other wealth.

Figure 2.2 shows different estimates of annual savings in Iceland from 1995-2011. The gross national savings as given by the national accounts shows a fairly steady decline from 17% of GDP in 1995 to 14% in 2009 and 11% in 2010 and 14% in 2011. This series has been compiled using the same methods for much longer time. The average was 26% during the 1970s and 19% during the 1980s which shows that in Iceland savings, measured in this way, as a share of GDP, have been on a declining trend for a long time. During this same period of time mandatory pension funds in Iceland developed from providing almost no pension to those that worked in the private sector to a situation where people in the private sector can expect to receive decent pension. This observation contradicts the claim that mandatory pension funds increase aggregate saving.

Figure 2.2
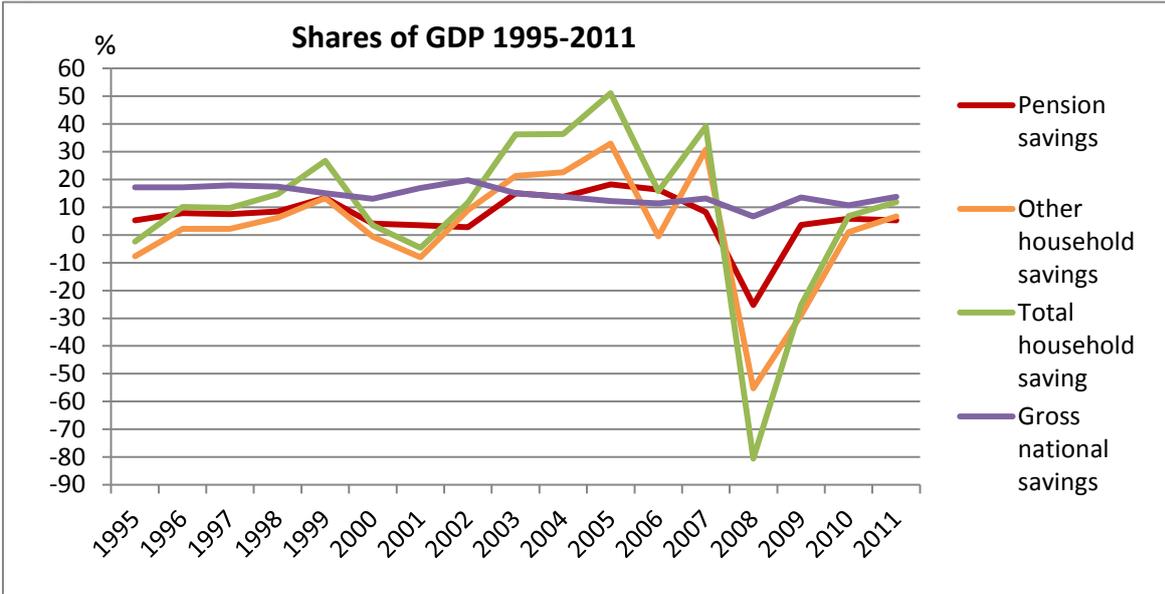
Sources: Statistics Iceland, the Central Bank of Iceland and the author's calculations. The estimates for gross national savings in 2008-2011 deviates from the ones given by Statistics Iceland as accrued interest income and cost of the failed banks on the income account has been subtracted.

The pension savings are estimated as the increase in the net assets of the pension funds at the average prices. The household savings are also estimated as the net increase in net assets of households. These series are very volatile. The total household savings peaked in 2005 when it was 51% of GDP and reached its lowest point in 2008 when it was –81% of GDP.

The national account data indicate that savings as a share of GDP have declined both before and after the assets of the pension funds started to increase. Other estimates based on current asset prices paint somewhat different picture. The volatility of these estimates is so large that it is not possible to make any conclusive statements about possible trends in the data.

Homeownership is very high in Iceland. For a long time it has been around 80%. Since 2000 Statistics Iceland has published the share of households in their consumption surveys that live in own homes and those that live in rented accommodation. This share was 79.9% during 2000-2002. Its highest value was 82.9% in 2003-2005. Since the collapse of the banking system in 2008 this share has declined and in 2008-2010 it was 75%.[6] These numbers can be compared to the share of homeownership in Ireland of 81%, 71% in the UK, 69% in USA, 66% in Finland, 55% in France, 52% in Denmark and 41% in Germany. The highest ratio given in Andrews e al. (2011) is in Spain where it measured 83% in 2004. Recently the Ministry of Welfare in Iceland has published data on homeownership. These data show a somewhat higher estimates for the share of households that

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[6] Data from Statistics Iceland's household expenditure surveys obtainable in Icelandic from Statistics Iceland's website, www.hagstofa.is.

own the accommodation where they live (see Velferðarráðuneytið, 2012) . These estimates show a peak of 85% in 2007 and a decline to 81% in 2009. Velferðarráðuneytið (2012) contains also data on owner-occupancy of low income households versus high income households. As expected the share of owner-occupancy is lowest among the poorest households. This indicates that in terms of the value of all accommodations the share that is owned by the occupants is probably well over 80%.

By the end of 2011 the value of the housing stock in Iceland amounted to 2,461 billion ISK. If we assume that 80% was owned by households the housing stock that households hold is worth some 1,969 billion ISK. The total private equity of households amounted to 1.564 billion ISK by the end of 2011, their debt amounted to 1,849 billion ISK and the net assets of the pension funds amounted to 2,097 billion ISK. The pension savings in Iceland by the end of 2011 exceeded households' other savings and constituted 59% of households' total savings, including savings in pension funds. In 1988 savings in mandatory pension funds amounted to 12% of households' total savings. In the same year 71% of households' equity, including the pension assets, was invested in own homes. In 2011 this ratio had declined to 54%. The share of the value of the homes in households' personal wealth (excluding assets in the pension funds) increased from 80-90% in 1988 to 120-130% in 2011.

In Iceland the average value of homes has been 2.5-3 times the annual disposable income. The real house price declined by 33% from its peak in the fourth quarter of 2007 to its lowest point in the first quarter of 2010. A little more than 20 years earlier the real house price declined also by 32% from the peak in the third quarter of 1983 to the second quarter of 1986. This means that the average homeowner suffered capital losses from these large price shocks of roughly one year's disposable income. Some other countries, e.g. the USA since the onset of the subprime loan crisis, have experienced decline in the real house price of similar magnitude.

The problems of the home owning households in Iceland were compounded during these periods of decline in the real house price by a simultaneous decrease in real disposable income. From 2007-2010 the per capita real disposable income in Iceland declined by 28%. Households in many other countries experienced decrease in real disposable income but in most cases the decrease was much smaller than in Iceland. In the models below we simplify the discussion by assuming that there is no income risk but only price risk associated with volatility in the house price. It seems reasonable to expect that adding income risk which is positively correlated with the house price risk will aggravate the financial shocks and problems for the home owning household discussed in this paper but not alter the conclusions qualitatively.

It is relevant to note that positive correlation between disposable inome on the one hand and house prices and rents on the other will make renting relatively more attractive compared to owning a house as decline in the house price will add capital losses to declining disposable incomes of home owning households but at the same time decrease rent.

3. A model where longevity is the only source of risk

One motive for saving is to accumulate resources to meet unexpected expenses. One way to do this is for each household to accumulate sufficient savings to meet such expenses. Another way is to purchase insurance policies against such eventualities leaving the precautionary savings to the insurer. As the insurance company is able to diversify the risks involved by insuring many people, some of which will have to meet large expenses of a specific type while others will only have small or no such expenses, makes it possible to reduce the amount of savings required to meet the expenses. It is not difficult to see that if there is no income insurance for retirement, and each household has to save for its expected maximum retirement, the required saving is much larger than in the case where it is possible to buy income insurance so that the household receives pension for the whole retirement. In this section this fact is explained by a simple model where the actual risk involved has been simplified very much by assuming that the length of the individual's (household's) life is the only stochastic variable. The nature of this uncertainty is also extremely simple and assumed to be publicly known. Most households are assumed to live for n periods of time and all work for $n - 1$ periods. A fixed share, $1 - \rho$, dies after $n - 1$ periods, but a fixed share lives for n periods. The value of ρ is assumed to be known and all households are equally likely to die at the end of the $(n - 1)$th period of their lives. During the nth period households are unable to work and have to rely on their own savings, or some pension from the government, or from some pension fund.

Households wage incomes during $n - 1$ periods of their working life are $y_i, i = 1, 2, \ldots, n - 1$, and known when the household enters the labour market. The household maximizes its expected time-separable utility from consumption (c_i):

$$EU = \mathbb{E}\{\sum_{i=1}^{n} \beta^{i-1} U_i(c_i)\} = \sum_{i=1}^{n-1} \beta^{i-1} U_i(c_i) + \beta^{n-1} \rho U_n(c_n) \tag{3.1}$$

where U_i is a concave utility function, β is a discount factor and $\mathbb{E}$ is the expectations operator, subject to the budget constraint:

$$(1 + r)w_0 + \sum_{i=1}^{n-1}(1 + r)^{1-i}(y_i - c_i) - (1 + r)^{1-n}c_n \geq 0 \tag{3.2}$$

where w_0 is some original wealth (inheritance) and r is the rate of interest.

The first order conditions for maximization are:

$$\beta^{i-1} U_i'(c_i) - \lambda(1 + r)^{1-i} = 0, i = 1, 2, \ldots, (n - 1) \tag{3.3}$$

and $\qquad \beta^{n-1} \rho U_n'(c_n) - \lambda(1 + r)^{1-n} = 0 \tag{3.4}$

where λ is Lagrange-multiplier. The budget constraint in Equation (3.2) is also part of the first order conditions.

Assuming that the utility function is concave ensures that there exists a solution to this problem. We write this solution as $\tilde{c}_i, i = 1, 2, \ldots, n$.

After k periods the equity of this household is

$$eq_k = (1 + r)^k w_0 + \sum_{i=1}^{k}(1 + r)^{k-i}(y_i - \tilde{c}_i) \tag{3.5}$$

If there is no pension scheme in this economy each household has to save individually for the retirement in period n. It plans to consume $\tilde{c}_n$ in this period and therefore, according to the life-cycle hypothesis, saves just enough to be able to afford this consumption, i.e. so that:

$$eq_n = (1 + r)^n w_0 + \sum_{i=1}^{n-1}(1 + r)^{n-i}(y_i - \tilde{c}_i) = \tilde{c}_n \tag{3.6}$$

Time of death is the only source of risk in this model and the only reason for saving is to secure means to consume during retirement. It is also assumed that the probability of the time of death is known with certainty. All households plan their consumption so that the value of their equity in period n equals the planned consumption in this period. As the share, $1 - \rho$ of them, will not live to spend their accumulated savings this part of aggregate savings will be inherited by later generations. These generations will though plan for their consumption not knowing if they will live the n-th period of their lives. If Equation (3.6) gives the average (or expected) equity and average consumption during retirement the average inherited wealth is $(1 - \rho)eq_n$. As this is the only wealth that households receive, besides the wealth they accumulate through their savings, we have that:

$$(1 - \rho)eq_n = w_0 \tag{3.7}$$

Using (3.6) and (3.7) to eliminate eq_n and obtain a solution for w_0 gives:

$$w_0 = \frac{(1-\rho)}{1-(1-\rho)(1+r)^n} \sum_{i=1}^{n-1}(1 + r)^{n-i}(y_i - \tilde{c}_i) \tag{3.8}$$

If the economy is populated by identical households or a representative household that has children with identical incomes as the parents, and also identical preferences, it is possible to solve for the stationary state for this model. The representative household inherits wealth given by Equation (3.8) at the beginning of the first period and has to leave the same amount to future generations.

If we now introduce a pension scheme into this economy and assume that there is no cost of operating this scheme (as we have ignored the cost of organizing the other savings schemes in this economy) the managers of the pension funds, knowing that $(1 - \rho)$ of all households will die before they reach retirement, can decrease the contributions to the scheme without risking that they will not be able to pay the promised pensions. The first generation that goes through this new system will inherit the same amount as earlier generations but as they will pay less for securing their livelihood during retirement they will be better off than earlier generations. As this lucky generation will not leave any savings for later generations these later generations will enter the economy without any inheritance. The income they expect to be able to spend during their lives will be the income they earn during their working years, the interest they earn on their savings minus the pension contributions but plus the pension if they live until retirement. The expected income they will spend on consumption will therefore be exactly the same income as in the case where there is no pension scheme. If the pension scheme is organized so that the contributions are the same as the households would make if they were free to decide them, i.e. if the contributions reflected the preferences of households and are equal to the households' voluntary savings before the introduction of the pension scheme, $y_i - \tilde{c}_i$, the expected and the actual utility of the households will be exactly the same as in the case where there is no pension scheme. The only difference between the two economies is that there is less saving (capital) in the economy with the pension scheme.[7] The difference in savings is given by Equation (3.8). Increases in the probability of death, $1 - \rho$, increases w_0, and therefore increases the negative effect on savings from the income insurance included in the services of pension funds with defined benefits. Adding uncertainty concerning the length of the time that individuals receive pensions, and concerning estimates of the various probabilities involved, will also strengthen this negative effect.

The influence on savings from this pension or income insurance can be substantial in modern economies where working life spans some 40-50 years, expected retirement some 15-20 years, the probability of living until retirement is high but variation in longevity is considerable.

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[7] This conclusion has to be modified when effects of smaller savings and therefore smaller stock of capital on wages and the interest rate in the economy are considered.

In the discussion above it was assumed that contributions to the mandatory pension scheme and the pension received conformed exactly with households' voluntary savings for old age. If the mandatory pension scheme demands larger contributions than the households would have saved and the households are unable to maintain their optimal consumption by borrowing then the mandatory pension scheme can increase savings. There is though no guarantee that this increase in savings increases welfare.

4. The effects of risky house prices on savings

Housing is a necessity but most households can choose between renting and owning their homes. A number of factors may influence this decision. OECD (2011a) notes that "home ownership is an important dimension of individual well-being. It protects owners from fluctuations in rents and ensures families a stable and secure shelter." Stability and security of the access to the dwelling are certainly reasons for preferring homeownership but in this paper it will be shown that on balance risks stemming from fluctuations in house prices and rents will favour renting to homeownership. In the simple models discussed below it is the risk created by fluctuations in house prices that prevents that all households choose homeownership which is assumed to be otherwise the preferred option.

Home owning households are free to modify their home to some extent that would usually be difficult or impossible if it was rented. There may also be financial reasons for preferring homeownership to renting as many governments give more subsidies to homeowners than renters. The usual form of subsidies for homeownership is to make some interest cost deductible from taxable income. State guarantees for special housing funds are also a form of subsidy. A subsidy for homeownership that is less often noted stems from the fact that the return on equity in the rented property is taxed in many countries, and this taxation affects the rent, while the return on the equity that homeowners own in their homes is not taxed. Finally, owners may take better care of their homes than renters, and in that way homeownership may be more efficient than renting. In this paper we will not try to model this but assume that households have to pay the full price for their homes both when they are homeowners and when they rent. If homeownership receives more subsidies than renting this can be included in the model by modifying the utility functions so that the utility households derive from living in an own home increases compared to the utility derived from renting the same accomodation.

The utility functions of a household in period k is $U_k(c_k, h_k, o_k)$ where o_k is a dummy taking the value 0 if the household rents but 1 if it is homeowner, while h_k and c_k are the volumes of consumption of housing services, and of other consumption, respectively. It is assumed that the following inequality is always valid:

$$U_k(c_k, h_k, 1) > U_k(c_k, h_k, 0) \qquad (4.1)$$

The utility function is assumed concave in c_k and h_k, and the indirect utility function, $\widetilde{U}_k(m_k, r \cdot ph_k, o_k)$, is assumed concave in $m_k = c_k + r \cdot ph_k$, the spending on consumption in period k. This last assumption is equivalent to the usual assumption in models where households optimize utility functions of one variable, wealth, income or aggregate consumption, over time.

To obtain some measures of the volatility in rental prices and house prices time series were obtained from the Macrobond data set. It was possible to obtain data for both price series for Denmark, Iceland, Ireland, Sweden and the UK. In all cases volatility of house prices exceeded that of rental prices by large margins. Different methods were used to measure volatility. In all cases real prices obtained by deflating the nominal prices by CPI were used. Table 4.1 shows the standard deviation of the ratio of real house prices and real rental prices and their respective trends obtained by using the Hodrick-Prescott (HP) filter. The table shows that according to this measure house prices

in Denmark were 12 times as volatile as the rental prices during the period 1997Q1-2010Q4. In Iceland the ratio was almost 3 and it was lowest in Sweden where the ratio was 2.4. In the period 1997Q1-2003Q4 this ratio was lowest in Iceland were it was 2.0 but highest in the UK where it was above 9.

Table 4.1

	Standard deviation of the ratio of real prices and HP-filtered trends (%)				
	1997Q1-2010Q4				
	Denmark	Iceland	Ireland	Sweden	UK
House prices	7.3	6.9	13.1	2.5	5.2
Rental prices	0.6	2.4	3.6	1.1	0.6
Ratio	11.9	2.9	3.6	2.4	8.8
	1997Q1-2003Q4				
House prices	3.0	5.4	12.2	2.2	4.8
Rental prices	0.4	2.8	2.9	0.8	0.5
Ratio	8.2	2.0	4.2	2.8	9.2

Source: Macrobond

Table 4.2 shows alternative measures of the volatility in the price series. For each country the maximum four-quarter increase and the maximum four-quarter decrease in the real house price and in the real rental price are shown. In all cases the maximum is larger in the case of house prices. The differences are large in all cases and in some cases very large.

Table 4.2

	Largest year-on-year increase and largest decrease in real house- and rental prices 1998Q1-2010Q4 (%)				
	Denmark	Iceland	Ireland	Sweden	UK
House prices					
Increase	27.3	25.1	26.6	12.6	23.9
Decrease	-20.1	-18.7	-16.0	-4.4	-19.0
Rental prices					
Increase	2.5	14.1	10.1	4.5	2.5
Decrease	-0.9	-4.3	-11.6	-1.6	-2.1

Source: Macrobond

Figure 4.1 shows the indices of real house price and the real rental price for Iceland. The figure shows that the volatility in the real house prices exceeds that in the real rental prices by a large margin.

Figure 4.1

Indices for Iceland 1997Q1-2010Q4, 1997Q1=100
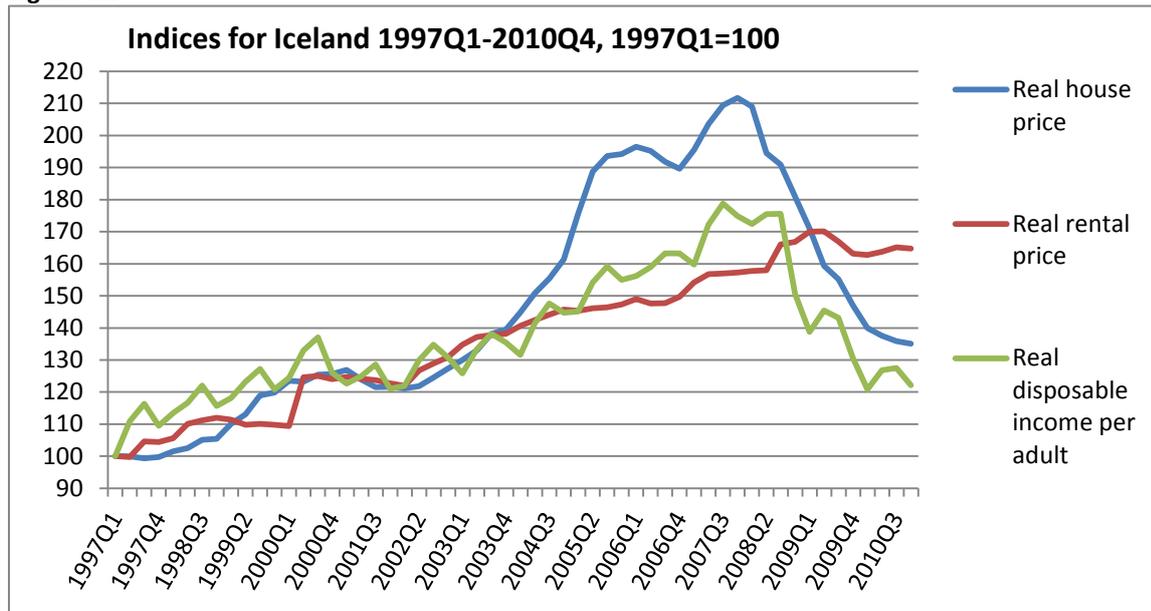
- Real house price
- Real rental price
- Real disposable income per adult

Real disposable income per adult person is also included in Figure 4.1. The correlation between real house price and real disposable income is very high (0.95). The correlation between year-on-year changes in the two series is also very high (0.85). The correlation between the real rental price and the real disposable income is positive but much lower. The correlation in levels is 0.69 but the correlation in year-on-year changes is only 0.28. These data show that shocks to households' income aggravate the problems for home owning households while the effects on renters are smaller.

In the model below it will be assumed that at the beginning of each period the households contract accommodation for the period either by purchasing a house at the prevailing market price or by renting. It will be assumed that houses last forever so there is no depreciation or maintenance cost. The only running cost of providing the accommodation is the capital cost. It will be assumed that the rental price is determined by arbitrage so that if the house price in period k is ph_k, rental price is $r \cdot ph_k$ where r is the rate of interest. As it is assumed that the rate of interest is constant, this means that rental prices are as volatile as house prices and they are perfectly correlated. This means that the volatility of the rent is assumed larger than what the empirical evidence discussed above indicate. It also means that the correlation between house prices and rents are assumed larger than the data show.

The assumption that the rental price is perfectly correlated with the house price can be justified on the basis that it simplifies the derivations of results from the model. It can also be justified on the basis of economic theory as it is the rental price which adjusts immediately to the new equilibrium where no arbitrage between the house price and the rental price is possible. Making alternative assumptions about the stickiness of the rental price, that are more in line with the data that were presented above, will make renting relatively more attractive as the price volatility is smaller, but it is unlikely to alter the main conclusions in this paper.

As the house price is stochastic it may change during period k causing capital gains or loss for homeowners. The renters will not have to worry about possible capital losses but the rental price in future periods is uncertain.

In this section, as in the previous section, it will be assumed that there is no reason for lenders to demand collaterals against their loans as the borrowers never default. It is therefore possible to write the household's budget constraint for period k as:

$$eq_{k+1} = (1 + r)eq_k + y_k - c_k + [o_k(ph_{k+1} - ph_k) - r \cdot ph_k]h_k \tag{4.2}$$

where eq_k is the personal equity or the net wealth of the household by the beginning of period k and end of period $k - 1$. It is assumed here that accommodation and other consumption is contracted at the start of the period but paid at the end of it.

It is assumed that the expected house price by the end of period k is equal to the house price by the beginning of the period, i.e. $\mathbb{E}_k\{ph_{k+1}\} = ph_k$ where $\mathbb{E}_k$ is the expectation operator giving the expected value of the variable on the basis of information available at the beginning of period k. It is then possible to write:

$$ph_{k+1} = (1 + \varepsilon_k)ph_k \tag{4.3}$$

where ε_k is a stochastic variable with mean $\mathbb{E}_k\{\varepsilon_k\} = 0$ and variance $\mathbb{E}_k\{\varepsilon_k^2\} = \sigma_k^2$. Using this to rewrite (4.2) gives the budget constraint in period k as:

$$eq_{k+1} = (1 + r)eq_k + y_k - c_k + [o_k\varepsilon_k - r]ph_k h_k \tag{4.2'}$$

The household is assumed to choose consumption of c_k and h_k so as to maximize expected discounted utility. The Bellman equation for the household's maximization problem is:

$$V_k(eq_k, ph_k) = \max_{c_k, h_k, o_k}\{\mathbb{E}_k\{U_k(c_k, h_k, o_k) + \beta V_{k+1}(eq_{k+1}, ph_{k+1})\}\} \tag{4.4}$$

where V_k is the value function for the sum of the household's discounted expected utility from optimal choices of consumption in period k and later period. At the start of period k $U_k(c_k, h_k, o_k)$ is non-stochastic so Equation (4.4) can be written as:

$$V_k(eq_k, ph_k) = \max_{c_k, h_k, o_k}\{U_k(c_k, h_k, o_k) + \beta\mathbb{E}_k\{V_{k+1}(eq_{k+1}, ph_{k+1})\}\} \tag{4.4'}$$

The first order conditions for maximum are:

$$\frac{\partial V_k}{\partial c_k} = \frac{\partial U_k}{\partial c_k} + \beta\mathbb{E}_k\left\{\frac{\partial V_{k+1}}{\partial eq_{k+1}}\frac{\partial eq_{k+1}}{\partial c_k}\right\} = \frac{\partial U_k}{\partial c_k} - \beta\mathbb{E}_k\left\{\frac{\partial V_{k+1}}{\partial eq_{k+1}}\right\} = 0 \tag{4.5}$$

and

$$\frac{\partial V_k}{\partial h_k} = \frac{\partial U_k}{\partial h_k} + \beta\mathbb{E}_k\left\{\frac{\partial V_{k+1}}{\partial eq_{k+1}}\frac{\partial eq_{k+1}}{\partial h_k}\right\} = \frac{\partial U_k}{\partial h_k} - \beta\mathbb{E}_k\left\{\frac{\partial V_{k+1}}{\partial eq_{k+1}}[r - o_k\varepsilon_k]ph_k\right\} = 0 \tag{4.6}$$

The assumption that the utility function is concave in aggregate spending ensures that the value function is concave in equity or wealth and that there is a solution to this optimization problem.

If the household is a renter and $o_k = 0$, the stochastic part of Equation (4.6) is only $-\beta r \cdot ph_k\mathbb{E}_k\left\{\frac{\partial V_{k+1}}{\partial eq_{k+1}}\right\}$, but if the household is a homeowner and $o_k = 1$, the stochastic part is the sum of two terms: $-\beta r \cdot ph_k\mathbb{E}_k\left\{\frac{\partial V_{k+1}}{\partial eq_{k+1}}\right\}$ and $\beta \cdot ph_k\mathbb{E}_k\left\{\frac{\partial V_{k+1}}{\partial eq_{k+1}}\varepsilon_k\right\}$. As explained in Appendix A below it is to be expected that the latter term is larger which shows that even if we assume that the rental price changes in the same way as the house price the effect of the price risk on the behaviour of homeowners is larger than the effect on the renter.

As V_{k+1} is a function of ph_{k+1} and therefore of ε_k, the price uncertainty has two different transmission mechanisms. Firstly, the price risk causes capital gain or loss on investments in housing amounting in period k equal to $(ph_{k+1} - ph_k)h_k = \varepsilon_k ph_k \cdot h_k$. Secondly, the price risk causes uncertainty concerning future cost of accommodation. For homeowners the second type of risk is negatively correlated with the first type. If the price increases, and they earn capital gain on their investment, the cost of accommodation, at least in the next period, will be higher and vice versa if the price decreases and they suffer capital losses. The renters will not earn any capital gains or suffer capital losses from the changes in the house price in the period but uncertainty about future price of accommodation may affect their behaviour. The sign of this effect is though uncertain and probably small. The homeowners, on the other hand, will decrease their consumption of housing (buy less expensive houses) and decrease their total consumption measured as $c_k + r \cdot ph_k h_k$. This means that risky house prices induce increased (precautionary) savings by homeowners. It is also probable that the savings of homeowners increases with the variance of the house price. Of the two types of mechanisms affecting the behaviour of homeowners it is the risk of capital losses or gains that is the largest. These contentions are explained and argued in greater detail in Appendix A below.

If households' incomes are positively correlated with the house price, and with rent, capital losses from homeownership aggravate the decline in income of home owning households while households that rent are compensated by lower rent in the next period.

If mandatory pension scheme is introduced into the model in this section the effects depend on how well the scheme reflects the planned savings for old age by the households, the efficiency of the financial markets in providing funding for responsible spending by households and on the regulations concerning collaterals and bankruptcies. In this section all borrowers have been assumed to pay back any loan they obtain in full and therefore there is no need for collaterals and there are no bankruptcy costs. In this section it has also been assumed that the households can always finance their optimal consumption plan at the given market rate of interest. Given these assumptions a mandatory pension scheme that is introduced into the economy, and replaces a private scheme with identical pension insurance, will not change the consumption plan of any household. Even if the payments into the mandatory pension scheme or the pension payments will not conform to the voluntary payments into the previous scheme the households will be able to finance their optimal plan in the efficient financial markets. If we remove these assumptions and allow that there may be reasons for lenders to demand collaterals, and that the financial markets may not be efficient, the introduction of a mandatory pension scheme will affect consumption and saving. If the payments into the mandatory pension scheme exceeds what the households would have done voluntarily, and the households cannot obtain finance for their optimal consumption plans, the outcome will be an increase in total savings and decrease in homeownership. Relaxing the assumption that all borrowers are responsible and there is no need to demand collaterals against loans or threaten with bankruptcies will also affect savings and homeownership as will be discussed in the next section.

5. The effects of collaterals and mandatory pension schemes on savings

In the real world, lenders demand collaterals to protect themselves against the possibility of default by borrowers. The borrower can declare bankruptcy whereby his debt is written off but there are also some costs involved such as higher rates of interest on future debt or impossibility of obtaining finance and impossibility of owning assets, e.g. an own home. In most cases there are also some social costs associated with bankruptcies.

To model bankruptcy realistically it must involve expected net cost to households. In the model in the previous section it is possible to assume cost of bankruptcy in the form of an higher rate of interest after bankruptcy or by assuming that the household is forced to rent for a number of

periods after bankruptcy which involves cost if homeownership is preferred to renting and possibly very much so. To keep the model as simple as possible it is assumed here that there exists some penalty for bankruptcy which can be calculated as a positive cost in the same period.

It is assumed that there is some mandatory pension scheme operated in the economy. In the case where there is no bankruptcy the budget constraint for period k is:

$$eq_{k+1} = (1 + r)eq_k + y_k - pi_k + po_k - c_k - [r - o_k \varepsilon_k]ph_k \cdot h_k = eq^1_{k+1} \qquad (5.1)$$

where pi_k is payment into the mandatory pension fund while po_k is pension. Usually $po_k = 0$ if $pi_k > 0$ and $pi_k = 0$ if $po_k > 0$. eq^1_{k+1} is the equity of the household by the end of period k if there is no default.

In the case where the household becomes bankrupt in period k the budget constraint is:

$$eq_{k+1} = (1 + r)eq_k + y_k - pi_k + po_k - c_k - [r - o_k \varepsilon_k]ph_k \cdot h_k$$

$$-\omega_k(y_k, (1 + o_k \varepsilon_k)ph_k \cdot h_k) = eq^0_{k+1} \qquad (5.2)$$

where ω_k is a function which gives the cost of bankruptcy which is payable in the same period. eq^0_{k+1} is the equity of the household by the end of period k if the household defaults in this period. Bankruptcy takes place if the equity before bankruptcy costs (eq^1_{k+1}) falls below a certain level. This level (θ_k) is assumed to be a function of income in the period and the value of the home at the end of the period. The household is assumed to become bankrupt in period k if:

$$(1 + r)eq_k + y_k - pi_k + po_k - c_k - [r - o_k \varepsilon_k]ph_k \cdot h_k \leq \theta_k(y_k, (1 + \varepsilon_k)ph_k \cdot h_k) \quad (5.3)$$

The left hand side of Equation (5.3) is equal to eq^1_{k+1} in Equation (5.1), i.e. the household's equity at the end of period k if there is no bankruptcy. Simple versions of the function θ_k are: $\theta_k(y_k, o_k(1 + \varepsilon_k)ph_k \cdot h_k) = 0$, $\theta_k(y_k, o_k(1 + \varepsilon_k)ph_k \cdot h_k) = \theta_{k,1}y_k$ and $\theta_k(y_k, o_k(1 + \varepsilon_k)ph_k \cdot h_k) = \theta_{k,2}o_k(1 + \varepsilon_k)ph_k \cdot h_k$ where $\theta_{k,1}$ and $\theta_{k,2}$ are constants. This formulation allows that those that rent their home ($o_k = 0$) can become bankrupt but the model in this paper does not contain mechanisms that might make some renters risk bankruptcy.

We will assume that it is possible to solve the inequality in (5.3) and obtain a solution of the form:

$$\varepsilon_k \leq \varepsilon^b_k = \varphi_k(c_k, ph_k, h_k, o_k, y_k, eq_k, pi_k, po_k) \qquad (5.4)$$

Equation (5.4) shows which values of ε_k are such that it will bankrupt the household. These limits, and the probability that ε_k is so low, are assumed to be known. The household takes a calculated risk of becoming bankrupt because homeownership brings sufficiently greater utility so that expected utility is higher than the expected utility from renting. The extent that the household is willing to take on risk of bankruptcy depends on the cost of bankruptcy. If these costs are extremely high the household will not take on any risk of bankruptcy, but if these costs are small the household will not bother to take much precautionary measures to diminish the probability of bankruptcy. If F_k is the distribution function for ε_k, $F_k(\varepsilon^b_k)$ gives the probability of bankruptcy.

The transition function for ph_k is the same as in Equation (4.3) in the previous section, the transition function for eq_k is now given by Equations (5.1)-(5.3) and the Bellman function for the household's maximization problem is the same as in Equation (4.4). Note though that the value function for the household is discontinuous at the value for $eq_{k+1} = \theta_k(y_k, (1 + \varepsilon^b_k)ph_k \cdot h_k)$ if the household can become bankrupt. The expected value, based on information available at the start of period k, of the value function by the start of period $k + 1$ is then:

$$\mathbb{E}_k\{V_{k+1}(eq_{k+1}, ph_{k+1})\} = \int_{-\infty}^{\varepsilon_k^b} \mathbb{E}_{k+1}\{V_{k+1}(eq_{k+1}^0, ph_{k+1})\}dF_k$$

$$+ \int_{\varepsilon_k^b}^{\infty} \mathbb{E}_{k+1}\{V_{k+1}(eq_{k+1}^1, ph_{k+1})\}dF_k \tag{5.5}$$

It is assumed that the function, $\mathbb{E}_{k+1}\{V_{k+1}(eq_{k+1}, ph_{k+1})\}$, is continuous and differentiable in c_k and h_k even if V_{k+1} is discontinuous. In this case the first order conditions become:

$$\frac{\partial U_k}{\partial c_k} = \beta \left[\int_{-\infty}^{\varepsilon_k^b} E_{k+1}\left\{ \frac{\partial V_{k+1}(eq_{k+1}^0(,,...,c_k,h_k,\varepsilon_k),ph_{k+1})}{\partial eq_{k+1}^0} \right\} dF_k \right.$$

$$\left. + \int_{\varepsilon_k^b}^{\infty} E_{k+1}\left\{ \frac{\partial V_{k+1}(eq_{k+1}^1(,,...,c_k,h_k,\varepsilon_k),ph_{k+1})}{\partial eq_{k+1}^1} \right\} dF_k \right]$$

$$+ \beta \left[\mathbb{E}_{k+1}\{V_{k+1}(eq_{k+1}^1(,,...,c_k,h_k,\varepsilon_k^b),ph_{k+1})\} \right.$$

$$\left. - \mathbb{E}_{k+1}\{V_{k+1}(eq_{k+1}^0(,,...,c_k,h_k,\varepsilon_k^b),ph_{k+1})\} \right] \frac{\partial \varphi_k}{\partial c_k} f_k(\varepsilon_k^b) \tag{5.6}$$

where $\varphi_{k,c} = \frac{\partial \varphi_k}{\partial c_k} > 0$, and f_k is the frequency function for ε_k, and

$$\frac{\partial U_k}{\partial h_k} = \beta \left[\int_{-\infty}^{\varepsilon_k^b} \mathbb{E}_{k+1}\left\{ \frac{\partial V_{k+1}(eq_{k+1}^0(,,...,c_k,h_k,\varepsilon_k),ph_{k+1})}{\partial eq_{k+1}^0} [r - o_k\varepsilon_k + \omega_{k,2}] \right\} dF_k \right.$$

$$\left. + \int_{\varepsilon_k^b}^{\infty} E_{k+1}\left\{ \frac{\partial V_{k+1}(eq_{k+1}^1(,,...,c_k,h_k,\varepsilon_k),ph_{k+1})}{\partial eq_{k+1}^1} [r - o_k\varepsilon_k] \right\} dF_k \right]$$

$$+ \beta \left[\mathbb{E}_{k+1}\{V_{k+1}(eq_{k+1}^1(,,...,c_k,h_k,\varepsilon_k^b),ph_{k+1})\} \right.$$

$$\left. - \mathbb{E}_{k+1}\{V_{k+1}(eq_{k+1}^0(,,...,c_k,h_k,\varepsilon_k^b),ph_{k+1})\} \right] \frac{\partial \varphi_k}{\partial h_k} f_k(\varepsilon_k^b) \tag{5.7}$$

where $\omega_{k,2}$ is the partial derivative of ω_k w.r.t. the second argument, $\omega_{k,2} = \frac{\partial \omega_k}{\partial[(1+o_k\varepsilon_k)ph_k \cdot h_k]} \geq 0.$

It is assumed that there exists a solution to Equations (5.6) and (5.7). Solving these equations gives the optimal consumption in period k, $\hat{c}_k$ and $\hat{h}_k$.

It is possible to obtain some results for the effects of a change in payments into mandatory pension schemes in period k. An increase in pi_k diminishes the equity of the household by the end of period k, both eq_{k+1}^1 and eq_{k+1}^0. The probability of default in the period has therefore increased, i.e. ε_k^b has increased. So even if we assume that this increase in pi_k replaces voluntary savings that the household would have done anyway, and the rules of the pension funds are such that it pays out pension exactly as the savings scheme that it replaces, the household is worse off than it was before. If the household continues to consume $\hat{c}_k$ and $\hat{h}_k$ in period k, leaving the left hand sides of the Equations (5.6) and (5.7) unchanged, the right hand sides have increased making it necessary to

search for new equilibrium values for $\hat{c}_k$ and $\hat{h}_k$.[8] For this to happen the left hand sides of the equations must increase and the right hand sides must decrease. This can only be achieved by consuming less and saving more.

The increase in savings will though be less than the increase in pi_k. This can be seen from Equations (5.6) and (5.7). If the household would increase its savings by the same amount as the incease in pi_k eq^1_{k+1} and eq^0_{k+1} would be no less than they were before, leaving the right hand sides equal to what they were before the increase in pi_k, while the left hand sides have decreased as consumption in period k is smaller and the utility function is concave. The solution must therefore be such that there is some increase in savings, but less than the increase in pi_k. This means that eq^1_{k+1} and eq^0_{k+1} will diminish when pi_k increases and the probability of default increases also as ε^b_k has increased.

The decrease in personal equity when pi_k increases means that the probability of default increases, both in period k, and in all later periods. For those that react to the increase in the risk of bankruptcy by going over to renting, the risk of bankruptcy will decline. How many households will go over to renting depends, among other things, on the availability and security of the rented accommodation. If homeownership is very much the preferred option households will continue to choose homeownership even if the risk of bankruptcy has increased significantly.

Equations (5.6) and (5.7) become identical to Equations (4.5) and (4.6) in the previous section in those cases where there is no risk of bankruptcy, i.e. where $F_k(\varepsilon^b_k) = 0$. The formulas show also that if there is no cost of bankruptcy, i.e. $\omega_k = 0$, changes in the pension contributions do not affect households' behavior. In this case $eq^0_{k+1} = eq^1_{k+1}$ and the right hand sides of Equations (5.6) and (5.7) become identical to the right hand sides of Equations (4.5) and (4.6). And if the cost of bankruptcy is so enormous that no-one risks bankruptcy (which they can avoid here by choosing to rent) then $\varepsilon^b_k = -\infty$ and $F_k(\varepsilon^b_k) = 0$ making the right hand sides of Equations (5.6) and (5.7) identical to the right hand sides of Equations (4.5) and (4.6).

It is possible to rewrite Equation (5.6) as

$$
\begin{aligned}
\frac{\partial U_k}{\partial c_k} = \beta \Bigg[& \int_{-\infty}^{\infty} E_{k+1} \left\{ \frac{\partial V_{k+1}\big(eq^1_{k+1}(,,\ldots,c_k,h_k,\varepsilon_k),ph_{k+1}\big)}{\partial eq^1_{k+1}} \right\} dF_k \\
& + \int_{-\infty}^{\varepsilon^b_k} E_{k+1} \left\{ \frac{\partial V_{k+1}\big(eq^0_{k+1}(,,\ldots,c_k,h_k,\varepsilon_k),ph_{k+1}\big)}{\partial eq^0_{k+1}} \right. \\
& \left. - \frac{\partial V_{k+1}\big(eq^1_{k+1}(,,\ldots,c_k,h_k,\varepsilon_k),ph_{k+1}\big)}{\partial eq^1_{k+1}} \right\} dF_k \Bigg] \\
& + \beta \Big[\mathbb{E}_{k+1}\big\{ V_{k+1}\big(eq^1_{k+1}(,,\ldots,c_k,h_k,\varepsilon^b_k),ph_{k+1}\big) \big\} \\
& - \mathbb{E}_{k+1}\big\{ V_{k+1}\big(eq^0_{k+1}(,,\ldots,c_k,h_k,\varepsilon^b_k),ph_{k+1}\big) \big\} \Big] \frac{\partial \varphi_k}{\partial c_k} f_k(\varepsilon^b_k) \qquad (5.6')
\end{aligned}
$$

and rewrite Equation (5.7) as

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[8] The concavity of the value function in equity ensures that the derivatives of it increase when the argument, eq^1_{k+1} and eq^0_{k+1}, decreases. The concavity of the value function also ensures that the change in its value diminishes when the argument, equity, increases. The decrease in $V_{k+1}(eq^1_{k+1}, ph_{k+1})$ when pi_k increases will therefore be smaller than the decrease in $V_{k+1}(eq^0_{k+1}, ph_{k+1})$ making their difference increase.

$$\frac{\partial U_k}{\partial h_k} = \beta \left[\int_{-\infty}^{\infty} \mathbb{E}_{k+1} \left\{ \frac{\partial V_{k+1}\big(eq_{k+1}^1(,,..,c_k,h_k,\varepsilon_k),ph_{k+1}\big)}{\partial eq_{k+1}^1} [r - o_k\varepsilon_k] \right\} dF_k \right]$$

$$+ \int_{-\infty}^{\varepsilon_k^b} \mathbb{E}_{k+1} \left\{ \frac{\partial V_{k+1}\big(eq_{k+1}^0(,,..,c_k,h_k,\varepsilon_k),ph_{k+1}\big)}{\partial eq_{k+1}^0} \right.$$

$$\left. - \frac{\partial V_{k+1}\big(eq_{k+1}^1(,,..,c_k,h_k,\varepsilon_k),ph_{k+1}\big)}{\partial eq_{k+1}^1} [r - o_k\varepsilon_k] \right\} dF_k \Bigg]$$

$$\int_{-\infty}^{\varepsilon_k^b} \mathbb{E}_{k+1} \left\{ \frac{\partial V_{k+1}\big(eq_{k+1}^0(,,..,c_k,h_k,\varepsilon_k),ph_{k+1}\big)}{\partial eq_{k+1}^0} \omega_{k,2} \right\} dF_k$$

$$+ \beta \big[\mathbb{E}_{k+1}\{V_{k+1}\big(eq_{k+1}^1(,,..,c_k,h_k,\varepsilon_k^b),ph_{k+1}\big)\}$$

$$- \mathbb{E}_{k+1}\{V_{k+1}\big(eq_{k+1}^0(,,..,c_k,h_k,\varepsilon_k^b),ph_{k+1}\big)\} \big] \frac{\partial \varphi_k}{\partial h_k} f_k(\varepsilon_k^b) \qquad (5.7')$$

Ignoring all but the first term on the right hand side of Equation (5.6') makes it identical to Equation (4.5) in the previous sections where there were no collateral constraints and no bankruptcies. Similarly, ignoring all but the first term on the right hand side of Equation (5.7') makes it identical to Equation (4.6). As the value function is increasing and concave, $eq_{k+1}^1(\varepsilon_k^b) > eq_{k+1}^0(\varepsilon_k^b)$, and ε_k^b and φ_k is an increasing function of both c_k and h_k, it can be shown that terms in Equations (5.6') and (5.7'), after the first ones, are non-negative. As the right hand side of Equations (5.6') and (5.7') increases the concavity of the utility function requires that the solutions, $\hat{c}_k$ and $\hat{h}_k$, are lower than in the case where these terms are zero.

It is important to keep in mind that it was assumed above that the increase in payments into the pension scheme and the pensions paid during retirement agree completely with what households would have done voluntarily. The increase in contributions to the pension scheme is then taken out of the household savings. If the increase in the pension scheme is a form of additional involuntary saving, the effects on total savings and consumption may differ from those discussed above. If the household is homeowner there is an incentive to increase savings as the risk of bankruptcy has increased. On the other hand, there is also an incentive to decrease savings in order to maintain as much as possible of the preferred consumption. This will lower the household's personal equity and increase the risk of bankruptcy.

6. Conclusions

It was shown above that mandatory pension schemes affect savings of individual households, homeownership and financial risk of lending to households. The effect on aggregate savings was found to be indeterminate. In Section 3 it was shown that mandatory pension schemes based on defined benefits provide income insurance that lead to a decrease in aggregate savings compared to what it would be if there was no such scheme or if the schemes were based on contributions. This feature can explain why aggregate saving, as it is defined in the national accounts, has declined in Iceland at the same time as the net assets of the mandatory pension funds have increased during the last 3-4 decades. In Section 4 it was shown that because of risks from volatile house prices homeownership induces households to increase their precautionary savings. The volatility of house prices also induce risk-averse households to rent rather than buy their accommodations that would

otherwise be the preferred option. In Section 5 it was shown that as increases in pension contribution diminishes the private wealth of households, excluding their pension wealth, the risk of bankruptcy of homeowners will increase. Savings of those households that remain homeowners after the increase in pension contributions will increase but, as some households may react to increases in the risk of bankruptcy by choosing to rent and their savings diminishes, the effect on aggregate savings is indeterminate. The financial risk associated with those households that will go over to renting diminishes. If the introduction of the mandatory pension schemes do not lead to a decrease in homeownership, aggregate savings increases and aggregate financial risk of lending to homeowners increases also.

The analysis of the effects of an increase in contributions to mandatory pension schemes was based on the assumption that it replaced voluntary savings and that payments out of the scheme agreed with the preferences of the households. If this is not the case, the households will search for ways to maintain some of their preferred consumption, diminishing further their personal equity and increasing the probability of default. This increase in the risk to homeownership will encourage some households to go over to renting.

In this paper we have ignored the risks involved in lending to firms that provide rented accommodation to households. If the risks in lending to these firms are greater than the risks in lending to households' purchases of own homes, as suggested by Gerlach (2012), mandatory pension savings that lead to decreases in homeownership may increase the overall financial risk in the economy even if the risks in lending to households decrease.

There has been some decline in homeownership in Iceland in recent years. The financial crisis in 2008 is probably the main reason for this decline. There are, so far, no clear indications that preferences for homeownership have declined. As shown in Section 2 above the savings of the households in Iceland, excluding their rights in the pension funds, has declined as a share of GDP compared to what it was in the 1970s, increasing the risk of bankruptcy.

The analysis in this paper explains that mandatory pension schemes with defined benefits may increase financial risk of lending to homeowners, an effect that as far as we are aware of has not been discussed in the literature. One way to diminish this negative effect of the mandatory pension schemes with defined benefits would be to limit the scope of these schemes to some basic pension and leave it to the households themselves to decide the additional pension savings that they would like to make. This arrangement can be defended on the grounds that what is needed to prevent insufficient savings for retirement by households is a pension scheme that provides basic income during retirement rather than one which intends to provide all income that both rich and poor households may demand during retirement. The provision of this minimum pension should take care of the moral hazard problem involved. Given that this basic pension is mandatory, the additional pension can be left to the individual households. Some households may want to purchase additional pension which promises a given income for the whole retirement while others may want to purchase additional pension where the rights to pension are defined on the basis of contributions or simply by accumulating personal saving. In some cases pension savings based on contributions can be used as a collateral as it is in essence a personal property of the household. This arrangement may not only add to welfare by allowing households to adjust their pension plans to their personal preferences, besides decreasing the risk of bankruptcy of homeowners, but may also create more diversified supply of pension schemes and limit some of the problems that few extremely large pension funds may create in capital markets. Gylfi Magnússon (2006) discusses these problems for the case of Iceland.

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Appendix A

The first order conditions in Equations (4.5) and (4.6) give that:

$$\frac{\partial U_k}{\partial c_k} = \beta \mathbb{E}_k \left\{ \frac{\partial V_{k+1}}{\partial eq_{k+1}} \right\} \tag{A.1}$$

and

$$\frac{\partial U_k}{\partial h_k} = \beta \mathbb{E}_k \left\{ \frac{\partial V_{k+1}}{\partial eq_{k+1}} [r - o_k \varepsilon_k] ph_k \right\} \tag{A.2}$$

First order Taylor-expansion around the solution to the deterministic case where $\varepsilon_k = 0$ in all states of nature, $\tilde{c}_k, \tilde{h}_k$, on the left hand side and first order Taylor-expansion around the solution to the deterministic case, $\widetilde{eq}_{k+1}$ and second order Taylor-expansion around $\varepsilon_k = 0$ on the right hand side give that:

$$\frac{\partial U_k}{\partial c_k} + \frac{\partial^2 U_k}{\partial c_k^2}(c_k - \tilde{c}_k) + \frac{\partial^2 U_k}{\partial h_k \partial c_k}\left(h_k - \tilde{h}_k\right) = \beta \frac{\partial V_{k+1}}{\partial eq_{k+1}} + \beta \mathbb{E}_k \left\{ \frac{\partial^2 V_{k+1}}{\partial eq_{k+1}^2}(eq_{k+1} - \widetilde{eq}_{k+1}) \right\}$$

$$+ \beta \mathbb{E}_k \left\{ \frac{\partial^2 V_{k+1}}{\partial eq_{k+1}\partial ph_k}(ph_{k+1} - ph_k) \right\} + 0.5\beta \mathbb{E}_k \left\{ \frac{\partial^3 V_{k+1}}{\partial eq_{k+1}\partial ph_k^2}(ph_{k+1} - ph_k)^2 \right\}$$

$$\Rightarrow \frac{\partial^2 U_k}{\partial c_k^2}(c_k - \tilde{c}_k) + \frac{\partial^2 U_k}{\partial h_k \partial c_k}\left(h_k - \tilde{h}_k\right) = \beta \frac{\partial^2 V_{k+1}}{\partial eq_{k+1}^2}[\mathbb{E}_k\{eq_{k+1}\} - \widetilde{eq}_{k+1}]$$

$$+\beta \frac{\partial^2 V_{k+1}}{\partial eq_{k+1}\partial ph_k}[\mathbb{E}_k\{ph_{k+1}\} - ph_k] + 0.5\beta ph_k^2 \frac{\partial^3 V_{k+1}}{\partial eq_{k+1}\partial ph_k^2}\mathbb{E}_k\{\varepsilon_k^2\}$$

$$= -\beta \frac{\partial^2 V_{k+1}}{\partial eq_{k+1}^2}\left[(c_k - \tilde{c}_k) + r \cdot ph_k\left(h_k - \tilde{h}_k\right)\right] + 0.5\beta ph_k^2 \frac{\partial^3 V_{k+1}}{\partial eq_{k+1}\partial ph_k^2}\mathbb{E}_k\{\varepsilon_k^2\}$$

$$\Rightarrow \left[\frac{\partial^2 U_k}{\partial c_k^2} + \beta \frac{\partial^2 V_{k+1}}{\partial eq_{k+1}^2}\right](c_k - \tilde{c}_k) + \left[\frac{\partial^2 U_k}{\partial h_k \partial c_k} + \beta r \cdot ph_k \frac{\partial^2 V_{k+1}}{\partial eq_{k+1}^2}\right]\left(h_k - \tilde{h}_k\right)$$

$$= 0.5\beta ph_k^2 \frac{\partial^3 V_{k+1}}{\partial eq_{k+1}\partial ph_k^2}\sigma_k^2 \tag{A.3}$$

and

$$\frac{\partial U_k(\tilde{c}_k,\tilde{h}_k)}{\partial h_k} + \frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial c_k \partial h_k}(c_k - \tilde{c}_k) + \frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial h_k^2}\left(h_k - \tilde{h}_k\right) =$$
$$\beta \mathbb{E}_k\left\{\frac{\partial V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}} + \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}^2}(eq_{k+1} - \widetilde{eq}_{k+1})[r - o_k\varepsilon_k]ph_k + \right.$$
$$\left.\frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}\partial ph_k}(ph_{k+1} - ph_k)[r - o_k\varepsilon_k]ph_k + 0.5\frac{\partial^3 V_{k+1}}{\partial eq_{k+1}\partial ph_k^2}(ph_{k+1} - ph_k)^2[r - o_k\varepsilon_k]ph_k\right\}$$

$$= \beta r \cdot ph_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}^2}(\mathbb{E}_k\{eq_{k+1}\} - \widetilde{eq}_{k+1}) - \beta o_k^2 ph_k^2 \cdot h_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}^2}\mathbb{E}_k\{\varepsilon_k^2\} -$$
$$\beta o_k ph_k^2 \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}\partial ph_k}\mathbb{E}_k\{\varepsilon_k^2\} + 0.5\beta ph_k^3 \frac{\partial^3 V_{k+1}}{\partial eq_{k+1}\partial ph_k^2}[r\mathbb{E}_k\{\varepsilon_k^2\} - o_k\mathbb{E}_k\{\varepsilon_k^3\}]$$

$$\Rightarrow \left[\frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial c_k \partial h_k} + \beta r \cdot ph_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}^2}\right](c_k - \tilde{c}_k)$$

$$+\left[\frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial h_k^2} + \beta r \cdot ph_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}^2}\right]\left(h_k - \tilde{h}_k\right) =$$
$$-\left[h_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}^2} + \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1},ph_k)}{\partial eq_{k+1}\partial ph_k}\right]\beta o_k ph_k^2 \sigma_k^2 + 0.5\beta ph_k^3 \frac{\partial^3 V_{k+1}}{\partial eq_{k+1}\partial ph_k^2}[r\sigma_k^2 - o_k\mathbb{E}_k\{\varepsilon_k^3\}] \tag{A.4}$$

Let

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} = \begin{bmatrix} \left[\frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial c_k^2} + \beta \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1})}{\partial eq_{k+1}^2}\right] & \left[\frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial h_k \partial c_k} + \beta r \cdot ph_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1})}{\partial eq_{k+1}^2}\right] \\ \left[\frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial c_k \partial h_k} + \beta r \cdot ph_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1})}{\partial eq_{k+1}^2}\right] & \left[\frac{\partial^2 U_k(\tilde{c}_k,\tilde{h}_k)}{\partial h_k^2} + \beta r^2 \cdot ph_k^2 \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1})}{\partial eq_{k+1}^2}\right] \end{bmatrix}$$

The assumption that the utility function and the value function are concave gives that d_{11} and d_{22} are both negative but the determinant $|D|$ is positive. Letting b_1 be the right hand side of (A.3) and b_2 the right hand side of (A.4) and using Cramer's rule to solve for $(c_k - \tilde{c}_k)$ and $\left(h_k - \tilde{h}_k\right)$ gives the solutions:

$$c_k - \tilde{c}_k = \frac{1}{|D|}\begin{vmatrix} b_1 & d_{12} \\ b_2 & d_{22} \end{vmatrix} = \frac{1}{|D|}[b_1 d_{22} - b_2 d_{12}] \tag{A.5}$$

$$h_k - \tilde{h}_k = \frac{1}{|D|}\begin{vmatrix} d_{11} & b_1 \\ d_{21} & b_2 \end{vmatrix} = \frac{1}{|D|}[b_2 d_{11} - b_1 d_{21}] \tag{A.6}$$

The sign of b_1 is not given by the assumption of concavity of the utility function but it should be relatively small when the price risk is small.

The concavity of the indirect utility function ensures that $\frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1}^2}$ is negative. The first term in the square brackets in the first term in b_2 is therefore positive but the second term is negative because $\frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1} \partial ph_k} = \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1}^2}(-r \cdot h_k)$ when $eq_{k+1} = (1+r)eq_k + y_k - c_k - r \cdot ph_k h_k$ as $\varepsilon_k = 0$. It follows that the first term in b_2 is:

$$-\left[h_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1}^2} + \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1} \partial ph_k} \right] \beta o_k ph_k^2 \sigma_k^2$$

$$= \left[-h_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1}^2} + \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1}^2} r \cdot h_k \right] \beta o_k ph_k^2 \sigma_k^2$$

$$= -(1-r)h_k \frac{\partial^2 V_{k+1}(\widetilde{eq}_{k+1}, ph_k)}{\partial eq_{k+1}^2} \beta o_k ph_k^2 \sigma_k^2 > 0$$

It follows that the direct effect of a change in the house price on the equity position of the household is stronger than the effect of the price change on future cost of housing. The sign of the second term in b_2 cannot be determined on the basis of the assumption of concavity of the utility function but it should be relatively small when the price risk is small.

Given these assumption we have from (A.5) that $c_k - \tilde{c}_k$ can be either positive or negative, but small, while (A.5) gives that $h_k - \tilde{h}_k$ is negative in most cases, because d_{22} is negative, while b_2 is positive and b_1 is relatively small.

Equations (A.5) and (A.6) give that:

$$c_k - \tilde{c}_k + r \cdot ph_k(h_k - \tilde{h}_k) = \frac{1}{|D|}[b_1 d_{22} - b_2 d_{12} + r \cdot ph_k(b_2 d_{11} - b_1 d_{21})]$$

$$= \frac{1}{|D|}[b_1(d_{22} - r \cdot ph_k d_{21}) + b_2(r \cdot ph_k d_{11} - d_{12})]$$

Concavity of the utility function ensures that $d_{22} - r \cdot ph_k d_{21}$ and $r \cdot ph_k d_{11} - d_{12}$ are both negative so that $c_k - \tilde{c}_k + r \cdot ph_k(h_k - \tilde{h}_k)$ must be negative if b_2 and $|D|$ are positive while b_1 is negligible.