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Should Icelandic pension funds hedge currency risk in their foreign investments?

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August 2014

Abstract

This paper discusses efficient hedging of currency risk in foreign investments through short term forward contracts. It is shown that for long term investors the efficient level of currency hedging depends on the behaviour of the exchange rate. If the exchange rate follows random walk the efficient hedging of the currency risk may be as large as it is for the short term investor, or possibly larger, but if the exchange rate follows a stationary stochastic process efficient hedging of the long term currency risk through short term forward contracts is zero in most cases. It is also shown that pass through of changes in the exchange rate into inflation forms a partial hedge for an investor that considers the real return and its volatility rather than the nominal return, diminishing the efficient level of forward currency contracts. It is shown that it is possible to use forward currency contracts for speculative investments. These contracts are profitable investment opportunities if the interest rate differential exceeds the expected change in the exchange rate. This same condition motivated the carry trade.

Keywords: Exchange rate risk, forward contract, hedging, long term, stationary process, mean reversion, pass through, speculative investment, carry trade

JEL Classification: F31, D81, G32

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1. Introduction

For some years before the collapse of the Icelandic banks in October 2008 Icelandic pension funds hedged the currency risk in their considerable foreign investments by buying short period forward contracts on Icelandic kronor. During 2008 when the Icelandic krona was depreciating rapidly the pension funds increased their hedging, leading to large losses on these contracts, both before the collapse of the banks, because of the depreciation of the krona, and also when the banks, which were counterparties in these contracts, went bankrupt. After the collapse of the banks these hedging practices of the Icelandic pension funds have been criticised. In its report Althing's Special Investigating Committee (SIC) notes that "it is noteworthy that the three pension funds that were investigated increased their currency hedges when the Icelandic krona depreciated which indicates that these funds, which should have long term investment objectives, were speculating in short term profits from the forex market." (SIC, 2010, Vol. 4, p. 134). The report quotes a British expert, Con Keating: "If the risk is to be hedged it should be done in the long term - that is multiple year. However the fund is mad if it takes the credit risk of a financial institution at these horizons. So the optimal route for the fund is to replicate the put option by trading arrangements. Short term hedging which is rolled over regularly is actually very expensive." (SIC, 2010, Vol. 4, p. 133) These statements are not explained further in the report.

Another important report focusing on the pension funds in the period before the collapse of the banks concludes that "it must be considered a serious mistake to increase hedging of the currency risk during the year (2008)." (Lífeyrissjóðaúttekt, 2012, p. 76) For some reason there are no arguments provided to support this statement, except that these increases in the hedging of the currency risk led to large losses in 2008. There are clear indications that increasing hedging of the currency risk in foreign investments when the krona had depreciated and decreasing it when the krona had appreciated was part of the pension funds' investment strategies as explained by three managers of large pension funds in an article in the daily morning paper, Fréttablaðið, November 4, 2010. In this article they write: "It is generally acknowledged that it is natural to diminish currency hedges when the exchange rate of the krona is very high and increase hedges when the exchange rate is low." (Guðmundsson e al., 2010) This statement implies that the managers are able to tell when the exchange rate of the krona is high and when it is low. If the exchange rate follows random walk, as some studies indicate, this is not possible. For it to be possible to tell when the exchange rate is high and when it is low the exchange rate must follow a stationary stochastic process and exhibit some mean reversion. It is possible to analyse the validity of this claim. It is also possible to examine the profitability of this hedging strategy, both theoretically and empirically, but it is incorrect to conclude that this strategy was a serious mistake because it led to large losses in 2008. That is to judge an investment strategy on the basis of what we know today about the outcome in one period, a mistake which is very common these days, but very unfortunate in a public report that should aim for educating the general public on matters which are relevant for its conclusions.

The pension funds have not replied directly to the criticism from SIC (2010), or Lífeyrissjóðaúttekt (2012), but they have reiterated several times that they think that their currency hedging was sound policy. In a report from a study group appointed by the pension funds and entitled "The pension funds' lessons from the collapse in 2008-2009" (Skýrsla Lærdómsnefndar, 2010) the conclusions concerning their hedging of the currency risk is that they "were natural instruments to diminish cycles and create stability in a portfolio. Their use was based on theoretical foundations and the experiences of foreign pension funds." No references are given for the theoretical foundations or the experiences of foreign pension funds.

In 2011 a news bulletin published by the Pension Fund for State Employees (Fréttabréf LSR, 2011) complained that the capital controls that were introduced in the autumn of 2008 prevented the fund from using currency hedges to avoid losses from the appreciation of the krona in 2010. The bulletin also tells of a survey of a group of European pension funds where it was found that 80% of the funds used some currency hedging. There is no mention of the extent of the currency hedging by those that use it, nor is any reason given why 20% of the funds in this group did not use any currency hedging. That some European pension funds use currency hedging does not prove that an Icelandic pension fund should use currency hedging. It depends on the special circumstances of each fund. It will be explained below that benefits of currrency hedging in the short period depend on the behaviour of the exchange rate, the pass through of changes in the exchange rate into inflation, and on the time until the foreign investments will be used for paying pensions. It is therefore quite possible that it is a bad policy for an Icelandic pension fund to hedge the currency risk in their foreign assets even though some pension fund in Belgium or in the UK might find such hedges to be efficient.

There exists large literature on currency hedging. Bailey and Stulz (1992), Kroner and Sultan (1993), and Amatatsu and Baba (2008) discuss currency hedging using different methods. Froot (1993) finds that currency hedging over long horizons are not as effective as currency hedging over short periods, and may even be counterproductive and increase the variance in the portfolio.

This paper's contribution is a theoretical discussion of hedging of currency risks by risk averse investors. Optimal hedging when the investor's utility is a function of the real value of the portfolio is discussed, and also the case of optimal hedging in the long term, both the case where utility is a function of the nominal value, and the case where it is a function of the real value of the portfolio. The theoretical discussion is simple enough to allow tractability and analytical results. Estimates of relevant theoretical relationships based on Icelandic data are used to provide practical examples.

In Section 2 the working of currency hedging through forward currency contracts in a one period framework is explained. To simplify the discussion it is assumed that the investor invests only in foreign assets and is therefore more dependent on the volatility of the exchange rate than most domestic investors, e.g. the Icelandic pension funds. It is shown that an investor buying currency hedge in a country where the interest rate is relatively high faces the same bet against the Uncovered Interest rate Parity (UIP) rule as the carry trader who is investing in the high interest rate currency. There are indications that the Icelandic pension funds' purchases of currency hedges were motivated more by speculative profits than by the reduction of risk.

Section 3 explains that if the rate of inflation reacts to changes in the exchange rate, the effects of changes in the exchange rate on the real value of foreign portfolios are mitigated by correlated changes in the price level, reducing the efficient hedging of this risk. Section 4 discusses stochastic processes for the exchange rate. It is shown that if the process is stationary future changes in the exchange rate are negatively correlated with the change in the present period and this negative correlation forms a hedge for the currency risk in the present period. The intuition behind this is that if the process is stationary, and the exchange rate appreciates in the present period, depreciations become more likely, and appreciations less likely, in the future. In that way the process "corrects" for what happens in the present period. The effects of a change in the exchange rate in the present period disappear gradually as time passes. For this reason the long term risk does not increase proportionally with time. Viewed differently, if the stochastic process is stationary, the correlation between the change in the exchange rate in the present period and the change in the exchange rate in the long term decreases towards zero when the long term is extended, which means that the relevance of hedging currency risk in the present period becomes smaller and eventually becomes zero. If the exchange rate follows random walk, the long term change in the exchange rate is correlated with the change in the present period and the variance increases proportionally in the long run as does the cost. For these reasons it is to be expected that if the exchange follows random walk the efficient hedging of the exchange rate risk in the present period is similar for long term investors as it is for short term investors.

Section 5 provides formal discussion of the efficient currency hedging in the present short period for a long term investor. It is shown that besides the stochastic characteristics of the exchange rate, the relative foreign and domestic returns influence the level of efficient hedging. For this reason there can be cases were the exchange rate follows a stationary stochastic process but the efficient level of currency hedging in the present period does not approach zero for the long term investor. This would though only happen if the stochastic process is close to random walk in the sense that the correlations between the change in the exchange rate in the present period and the change over the long period declines slowly. It follows that if the stochastic process is stationary, and there is sufficient mean reversion in the process, Keating's claim in SIC (2010) that currency hedging by rolling over short term contracts is too expensive is valid. Mean reversion in the exchange rate also provides a theoretical foundation for the speculative investment rule which says that purchases of currency hedges should be increased when the exchange rate of the krona is low and decreased when it is high. If the stochastic process is random walk, and the expected foreign returns equal to the expected domestic returns, the efficient hedging is independent of the time until the funds will be used. In this case Keating's claim is not valid and there is no theoretical foundation for the speculative investment rule. The formal arguments in Section 5 are based on the assumption that the relevant stochastic variables are log-normally distributed.

Section 6 discusses Icelandic data and estimates the relevant equations. Mean reversion in the real exchange rate of the Icelandic krona is estimated and pass through of changes in the exchange rate of the krona into inflation in Iceland. These two relationships imply that there is the same mean reversion in the nominal exchange rate towards a trend which is deterministic, but may also be stochastic, introducing a non-stationary element into the nominal exchange rate. These results provide some support for the belief that the three managers express in Guðmundsson e al. (2010) that they can tell when the exchange rate is high and when it is low and use that information to adjust their purchases of forward currency contracts, but it also follows that the optimal hedging of the currency risk in the present period is close to zero for long term investors like the pension funds. Section 7 concludes.

2. Currency hedging in the short term

The exchange rate of the Icelandic krona is volatile. Figure 1 shows changes in the exchange rate when it is measured as the price of foreign currency.[1]

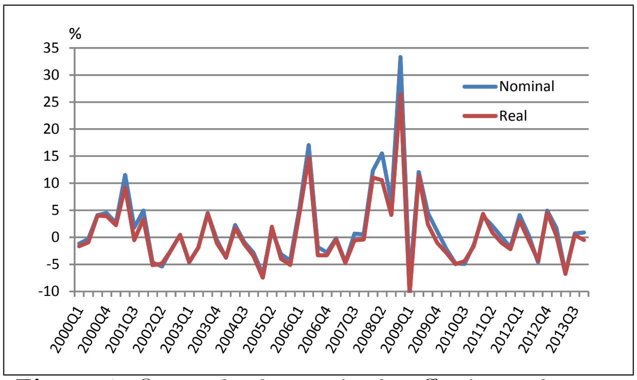

Figure 1. Quarterly changes in the effective exchange rate of the Icelandic krona 2000Q1-2013Q4. Source: Central Bank of Iceland.

For the period 2000Q1-2013Q4 the average quarterly change in the nominal rate was 1.42% while the average change in the real rate was 0.50%. The standard deviation of changes in the nominal exchange rate was 6.9% while the standard deviation of changes in the real rate was 6.0%. The effective exchange rate is the average of the exchange rate of the krona against several currencies. In most cases volatility of exchange rates of a currency is larger than the volatility of the index. The large volatility of the exchange rate of the Icelandic krona creates obvious incentives for risk averse investors, which the pension funds should be, to seek out methods to hedge this risk.

By the end of 2013 the total assets of the Icelandic pension funds amounted to 133% of the GDP in Iceland in 2013. Of these funds a quarter, or 33% of GDP in 2013, was invested abroad. (see sedlabanki.is, 2014) The share of foreign investment has declined from just below 30% before the introduction of the capital controls in 2008. These controls prevent the pension funds from investing new domestic funds abroad.

To simplify the discussion in this paper it is assumed that the investor invests only in foreign assets. This investor is therefore more exposed to exchange rate fluctuations than is the case for most Icelandic investors. It also means that the analysis does not take into account the reduction of risks that follows from diversification of an Icelandic portfolio by foreign investments. Taking domestic investments into account would make the effect of the exchange rate risk smaller in relation to the value of the portfolio, and therefore make

[1] This is the opposite of what is usually meant by the exchange rate of the Icelandic krona which is the price of the krona denominated in foreign currency. In the theoretical discussion in this paper the exchange rate of the krona is the one shown in Figure 1. Occasionally, when discussing appreciation and depreciation of the krona, it may refer to the proper meaning of these terms as changes in the price of the krona in terms of foreign currency. It is not always pointed out which definition is used but in those cases it should be clear from the context.

the investor more willing to take on this risk. It follows that for obtaining realistic result for this kind of investors, in a model where there are only foreign investments, one should use lower coefficient of relative risk aversion.

The analysis in this paper will be conducted with the help of formal models. Let W_t be the value of the foreign portfolio in domestic currency at the start of period t, ρ_t the rate of return on foreign investments, and s_t the nominal exchange rate expressed as the price of foreign currency in terms of domestic currency at the start of period t. Then s_{t+1} is the spot rate at the end of period t/beginning of period $t+1$ and the relative change in the exchange rate during period t is $s_{t+1}/s_t = 1 + e_t$. Using this notation the value of the foreign portfolio by the end of period t can be written as

$$W_{t+1} = (1 + \rho_t)(1 + e_t) W_t \tag{1}$$

The foreign return (ρ_t) and the relative change in the exchange rate (e_t) are assumed to be stochastic variables.

An investor who can borrow in foreign currency can hedge the exchange rate risk by financing the foreign investments abroad and investing own funds domestically. If the investor can borrow abroad at a risk free rate of R_t^f and deposit the money domestically at the risk free rate R_t^d, the value of the foreign investment part of his portfolio by the end of period t/beginning of period $t+1$ is

$$W_{t+1} = (1 + \rho_t)(1 + e_t) W_t - (1 + R_t^f)(1 + e_t) W_t + \left(1 + R_t^d\right) W_t \tag{2}$$

where the first term on the right hand side is the value of the foreign investment as in Equation (1), the second term is the cost of the foreign loan used to finance the foreign investment and the third and the last term is the return on the domestic account. Simplifying Equation (2) gives that

$$W_{t+1} = (\rho_t - R_t^f)(1 + e_t) W_t + \left(1 + R_t^d\right) W_t \tag{2'}$$

which shows that if these arrangements are made the risk in the exchange rate affects only part of the foreign return, but not the principal.

In most cases investors do not hedge against currency risks in their foreign investments by borrowing in foreign currency. Instead, the hedging is accomplished through contracts with financial institutions in the home country. These contracts can take different forms, but this paper discusses only forward currency contracts, which was the method for hedging currency risk that was commonly used by the Icelandic pension funds. In a forward currency contract the parties agree on the sum of money that should be hedged. This sum of money is changed into foreign currency (at least in theory) at the spot rate on the day when the contract is made. The contract also stipulates the exchange rate at some given time in the future when the contract expires and the sum of money in foreign currency is exchanged again into domestic currency. This future exchange rate, the so called forward rate, is fixed in the contract. This forward rate at the end of period t is denoted f_{t+1}. It is assumed that the investor buys currency hedge for $h \cdot W_t$ where h is a real number. The value of the foreign investment part of the investor's portfolio, which is the sum of the return on the foreign investment and the profits/losses from currency hedging, can now be written as

$$W_{t+1} = (1 + \rho_t)(1 + e_t) W_t + h \cdot W_t (f_{t+1} - s_{t+1}) / s_t \tag{3}$$

Simplifying gives that

$$W_{t+1} = (1 + \rho_t - h)(1 + e_t) \cdot W_t + h \cdot W_t \cdot f_{t+1}/s_t \tag{3'}$$

which shows how the currency hedging works by reducing the dependence of the value of the portfolio by the end of the period on the changes in the exchange rate by adding to the original portfolio an asset which is such that the changes in its value are negatively correlated with the changes in the value of the original portfolio.

A complete description of the forward contract requires the determination of the forward rate. Usually this rate is determined by the so called covered interest rate parity which gives the forward rate as the difference between a domestic interest rate and a foreign interest rate. If R_t^d is the domestic rate and R_t^f is the foreign rate then

$$\frac{f_{t+1}}{s_t} = \frac{1 + R_t^d}{1 + R_t^f} \tag{4}$$

The idea behind this formula is that if it is not valid it is possible to earn pure profits, i.e. some arbitrage is possible. If the forward rate is lower than Equation (4) prescribes an investor who can borrow abroad at a rate R_t^f, exchange the money into domestic currency at the spot rate today, invest the sum in an account giving the rate of interest of R_t^d, and then exchanges the money back into foreign currency at the forward rate f_{t+1}, makes a profit, and this profit is riskless as all variables in Equation (4) are assumed given by the beginning of the period. If the forward rate is higher than Equation (4) prescribes risk free profits can be earned by borrowing domestically at the rate R_t^d, exchanging the money into foreign currency, and invest abroad in an account with the rate R_t^f, and, by the end of the period, exchange the money back into domestic currency at the forward rate f_{t+1}.

Substituting Equation (4) into Equation (3) gives

$$W_{t+1} = (1 + \rho_t)(1 + e_t) W_t + h \cdot W_t \left(\frac{1 + R_t^d}{1 + R_t^f} - (1 + e_t) \right) \tag{5}$$

or, after simplifications,

$$W_{t+1} = (1 + \rho_t - h)(1 + e_t) W_t + h \cdot W_t \frac{1 + R_t^d}{1 + R_t^f} \tag{5'}$$

If the investor takes out a forward contract for 100% of the value of the foreign portfolio by the beginning of the period, i.e. if h = 1, changes in the exchange rate cause much smaller variations in W_{t+1} in Equation (5') than in Equation (1). When h = 1 Equation (5') becomes almost identical to Equation (2') above.

In theoretical discussions of forward currency contracts it is frequently used that if the variables ρ_t, e_t, R_t^d and R_t^f are small, the product of two of them is very small and the following version of Equation (5) is approximately correct

$$W_{t+1} = (1 + \rho_t) W_t + (1 - h) e_t W_t + h \cdot \left(R_t^d - R_t^f \right) W_t \tag{5''}$$

In Equation (5") there are no costs associated with the forward contract. It is difficult to obtain information about the cost of currency hedging of the Icelandic pension funds (See Lífeyrissjóðaúttekt, 2012, Vol. 1, pp. 75-78). Theoretically it is possible to assume that the costs are included in the interest rates, R_t^d and R_t^f, and many writers have implicitly done that. In this paper it will be assumed that there is a special cost associated with forward contracts. It is assumed to be c_t per unit of the sum of money being hedged. Including this cost in Equation (5”) gives

$$W_{t+1} = (1+\rho_t)\, W_t + (1-h)\, e_t W_t + h \cdot \left(R_t^d - R_t^f - c_t\right) W_t \tag{5”’}$$

It was shown above that an investor that invests abroad can diminish the exposure to exchange rate risk through forward currency contracts. The next question is how much the investor should protect his portfolio against currency risks. The answer to that question depends on how large the risk is, how risk averse the investor is, and on the cost of the forward contract. An investor who is very risk averse will buy forward contract for almost 100% of the value of the portfolio even if such contracts are costly. A less risk averse investor will take on more exchange rate risk to diminish the cost of the forward contracts. The simplest way to analyse this formally is to use Markowitz (1952) mean-variance utility function. The formal analysis in this section uses the utility function

$$E\{U\,(W_{t+1})\} = E\{W_{t+1}\} - \frac{\varphi}{W_t} Var\,(W_{t+1}) \tag{6}$$

where U denotes the utility function, E is the expectation operator and Var denotes the variance. φ/W_t is related to the coefficient of absolute risk aversion $(-U''\,(W)\,/U'(W))$ and therefore φ is related to the coefficient of relative risk aversion $(-W \cdot U''\,(W)\,/U'(W))$. It is possible to show (see e.g Sargent, 1979, pp. 154-155) that if the agent‘s utility function is given by $U(W) = -\exp(-\lambda W)$ where W stands for wealth, which is normally distributed, and λ is the absolute risk aversion, maximizing the expected value of this utility function is equivalent to maximizing $E\{W\} - 0.5\lambda Var(W)$. It follows that if W_{t+1} is normally distributed the utility function in Equation (6) has the absolute risk aversion close to $2\varphi/W_t$ and the relative risk aversion close to 2φ. In economic applications it is common to assume that relative risk aversion is above 1, but 1 is common in many practical applications, e.g. in many macro models. Many writers have assumed the value of 2, but numbers above 10 are considered very high. (See e.g. Lucas, 1987, p. 26). The value of $\varphi = 1$, giving relative risk aversion of 2, seems to be a reasonable middle of the road assumption.

As noted above ignoring domestic investments and the possible hedging of risks in the domestic portfolio by foreign investments changes the situation for risk-taking in foreign markets and therefore affects what is the suitable value for φ in the examples below. To take one extreme example, if the investor invests 70% of the portfolio in indexed bonds with fixed real interest rates issued by the Icelandic government (which is the largest type of assets in the Icelandic pension funds’ portfolios), the variance of the real value of the domestic part of the portfolio is zero and the variance of the real value of the total portfolio is equal to the variance of the real value of the foreign investments. In this case this risk would be spread over a portfolio which is 3.3 times the value of the foreign portfolio and affecting the relative risk aversion accordingly. This would obviously be an extreme case, but, more generally, it can be expected that the choice of a given level of risk aversion for the foreign portfolio only will make the investor more averse to risks in the foreign portfolio than would be the case if the domestic portfolio was included in the analysis.

Assuming that R_t^d and R_t^f are risk free rates while ρ_t and e_t are stochastic, and calculating expectations of the expression in Equation (5”), gives

$$E\{W_{t+1}\} = \left[1 + E\{\rho_t\} + (1-h)\, E\{e_t\} + h \cdot \left(R_t^d - R_t^f - c_t\right)\right] W_t \tag{7}$$

Calculating the variance gives

$$Var\left(W_{t+1}\right) = \left[-Var\left(\rho_t\right) + (1-h)^2 Var\left(e_t\right)\right] W_t^2 + 2\left(1-h\right) Cov\left(\rho_t, e_t\right) W_t^2 \tag{8}$$

Substituting from Equations(7) and (8) into Equation (6) and differentiating with respect to h gives the first order condition

$$\left[-E\{e_t\} + \left(R_t^d - R_t^f - c_t\right)\right] W_t - \frac{\varphi}{W_t}\left[-2\left(1-h\right) Var\left(e_t\right) - 2Cov\left(\rho_t, e_t\right)\right] W_t^2 = 0 \tag{9}$$

Solving for the optimal value of h gives

$$h^* = 1 + \frac{R_t^d - R_t^f - E\{e_t\} - c_t}{2\varphi Var\left(e_t\right)} + \frac{Cov\left(\rho_t, e_t\right)}{Var\left(e_t\right)} \tag{10}$$

It seems reasonable to expect the correlations between the rate of return on foreign investments and changes in the exchange rate of the Icelandic krona to be small, and probably insignificant, so the last term in Equation (10) should be close to zero. The denominator in the middle term consists of two factors, the coefficient of risk aversion (2φ) and the variance of the changes in the exchange rate. If the nominator is negative, efficient hedging increases with the risk aversion and the exchange rate risks, measured by the variance. If, on the other hand, the nominator is positive, which means that the expected cost of forward currency contracts is negative, it becomes optimal to hedge more than 100% of the currency risk, i.e. it becomes profitable to make speculative investments in currency contracts. In this case Equation (10) gives that larger risk aversion and larger exchange rate risks lead to smaller investments in currency contracts.

The interest rate differential, $R_t^d - R_t^f$ in Equation (10), can be quite large, and sometimes such large interest rate differentials lead to large inflow of capital into a country where the rate of interest is high, causing the exchange rate to appreciate. In such situations the expected change in the exchange rate, $E\{e_t\}$, can be negative for a while making the nominator in Equation (10) still more positive, and investments in forward currency contracts still more profitable.

The difficulty with using Equation (10) is that the exchange rate is very volatile. There does exist a widely used formula that predicts that the nominator in the middle term in Equation (10) should be near zero, or slightly negative. The UIP rule predicts that the expected change in the exchange rate should be equal to the short term interest rate differential, i.e. $R_t^d - R_t^f - E\{e_t\}$ should be zero. The argument for this rule is simply that if markets are efficient the expected rate of return should be the same in all countries where capital flows freely.

Table 1 below gives the values of the optimal currency hedge if UIP is valid and $c_t = 0.0025$ per quarter. This cost is based on information in Chou Associates Fund (2007). The table shows the optimal hedge for three different values for the relative risk aversion, from 1 when $\varphi = 0.5$ to 4 when $\varphi = 2.0$, and three different values for the exchange rate risk. The first column uses the variance of quarterly changes in the price of the euro for the period 2000Q1-2007Q4, the second uses the variance for the period 2000Q1-2008Q3, and the third uses the variance for the period 2000Q1-2008Q4.

$\varphi\backslash$**Var** (e_t)	*0.00235*	*0.00347*	*0.00563*
0.5	-0.064	0.279	0.556
1.0	0.468	0.640	0.778
2.0	0.734	0.820	0.889
Table 1	Optimal	hedging	(h^*)

Table 1 shows that the value of h^* varies a lot depending on the assumptions, from 0.889 when the risk aversion is high ($\varphi = 2$) and risk is high ($Var(e_t) = 0.00563$) to -0.064 when risk aversion is low ($\varphi = 0.5$) and risk is low ($Var(e_t) = 0.00235$). The negative number means that in this case it is actually optimal for the domestic investor not to hedge the exchange rate risk of the foreign investement, but to take on additional exchange rate risk. The remuneration for this risk taking, 0.0025 per quarter, exceeds the cost of taking on additional exchange rate risk in this case.

If $R_t^d - R_t^f - E\{e_t\} - c_t = 0$ and the variation in foreign returns is independent of changes in the exchange rate so that the last term in Equation (10) is also zero, then $h^* = 1$, i.e. it is optimal to hedge all of the currency exposure from the foreign investments. If $R_t^d - R_t^f - E\{e_t\} - c_t > 0$ it becomes optimal to buy forward contracts for more than 100% of the currency exposure caused by the foreign investments because in this case purchasing forward currency contracts is expected to be a profitable business in itself.

Interest rates in Iceland have been high compared to interest rates abroad. Figure 2 shows the difference in short term interest rates in Iceland and in the Euro Area and the annualised change in the nominal exchange rate. The interest rate differentials in 2007 and 2008 have been adjusted for the 3-month spread on swaps (see discussion in Daníelsson, 2010). The changes in the exchange rate are the changes in the average price of the euro between consecutive quarters. The figure also shows the changes in the exchange rate index between consecutive quarters. 2008Q4, when the price of the euro increased by 187% on an annual basis, and the exchange rate index increased by 216%, is not included in the figure!

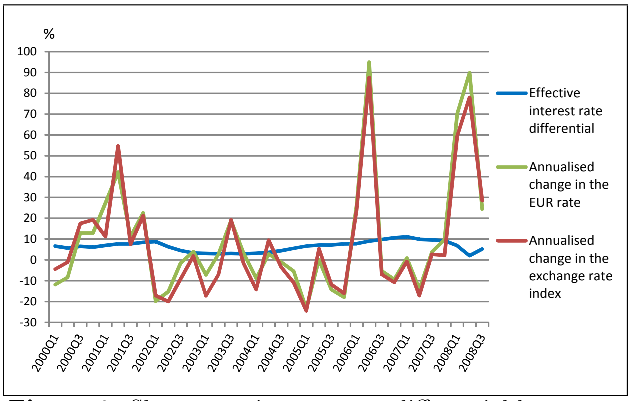

Figure 2. Short term interest rate differential between Iceland and the Euro Area and annualised changes in the exchange rate 2000Q1-2008Q3. Source: Central Bank of Iceland.

The average interest rate differential for the period from 2000Q1-2008Q3 was 6.5% with a standard deviation of 2.5%. The average annualised change in the price of the euro was 9.0% during the same period with a standard deviation of 28.1%. Adding the change in the exchange rate in the fourth quarter of 2008 the average annualised change in the price of the euro was 14.0% and the standard deviation 40.6%. Within the nine year period from 2000 to 2008 there was a period of four years, 2002-2005, where the interest rate differential was almost always larger than the change in the exchange rate and there were substantial profits to be made from buying forward currency contract for the Icelandic krona. During this period the existing evidence on the discussion of currency hedging among managers of pension funds in Iceland shows strong focus on the profit opportunity arising from the interest rate differentials being larger than the expected increase in the price of foreign currency (see Samúelsdóttir, 2004, and Tryggvason and Tryggvason, 2004). This is exactly the same speculation as foreign investors involved in the carry trade were making at the same time.

The investor that hedges against the changes in the euro rate will profit if the interest rate differential is larger than the increase in the euro rate, i.e. when the blue line in Figure 2 is above the green one (or rather above a line which is 1% below the blue line if the annualised cost of forward currency contracts is 1%). If the price of the euro increases more than the interest rate differential, i.e. if the green line is above the blue one, there are losses from the currency contracts.

Icelandic investor with some foreign assets in the portfolio has to balance the considerable currency risk against the expected profit/loss from the contract using the appropriate level of risk aversion. Table 2 shows optimal currency hedging obtained from Equation (10), using different assumptions concerning the interest rate differentials, the expected change in the exchange rate, and the currency risk, based on data for different periods of time. Two different values for the relative risk aversion are used

Period	$R_t^d - R_t^f$	$E\{e_t\}$	$Var(e_t)$	$h^*(\varphi = 0.5)$	$h^*(\varphi = 2)$
2000Q1-2008Q3	0.0162	0.0166	0.0035	0.16	0.79
2000Q1-2008Q4	0.0160	0.0246	0.0056	-0.96	0.51
2002Q1-2005Q4	0.0125	-0.0145	0.0008	30.63	8.41
2002Q1-2006Q4	0.0147	-0.0014	0.0028	5.86	2.21

Table 2

If the estimates of the relevant variables are based on data for the period 2002Q1-2006Q4 investing in forward contracts was hugely profitable and obviously quite tempting. If 2002Q1-2005Q4 are used as a reference, investing in these contracts was still more profitable and tempting speculative investment. If data for the period 2000Q1-2008Q3 are used the cautious investor with $\varphi = 2$ purchases forward currency contracts for 79% of the foreign investment while the moderately cautious investor buys for 16%. If data for 2008Q4 is added the cautious investor purchases forward currency contracts for 51% and the moderately cautious investor does not purchase any forward currency contracts, but is tempted to invest in the selling of such contracts.

It is possible to solve the problem of the optimal purchase of forward currency contracts using the exact formula in Equation (5) rather than the approximate formula in Equation (5"), and using the utility function $U(W) = -\exp(-2\varphi W)$ rather than the mean-variance formula in Equation (6). Table 3 below gives the outcomes (the first column and the third are the same as in Table 2 for convenience).

Period	Mean-variance $\varphi = 0.5$	$\varphi = 1$	$\varphi = 2$	$U(W) = -\exp(-2\varphi W)$ $\varphi = 0.5$	$\varphi = 1$	$\varphi = 2$
2000Q1-2008Q3	0.16	0.58	0.79	0.11	0.54	0.76
2000Q1-2008Q4	-0.96	0.02	0.51	-1.29	-0.16	0.41
2002Q1-2005Q4	30.63	15.82	8.41	23.92	14.48	6.64
2002Q1-2006Q4	5.86	3.43	2.21	5.17	3.09	2.05
Table 3	Optimal	hedging	(h^*)			

Generally the value of h^* is a bit lower when the exact formula and the exact distribution are used rather than the approximations. The differences are small. Further investigations show that the approximations used to get Equation (5") from Equation (5') do not lead to significant differences, but the use of the mean-variance function instead of the exponential utility function and ignoring the effects of possible skewness and kurtosis of the distribution have larger impact. The distribution of changes in the price of the euro from 2000Q1-2008Q3 has skewness of 1.37 and kurtosis of 1.87. If the change in the price of the euro in 2008Q4 is added, the skewness becomes 1.97, and the kurtosis 4.70. Figure 3 shows the frequency distribution of the quarterly changes in the price of the euro for the period 2000Q1-2008Q4, and the frequency distribution of a normal distribution with the same mean and variance.

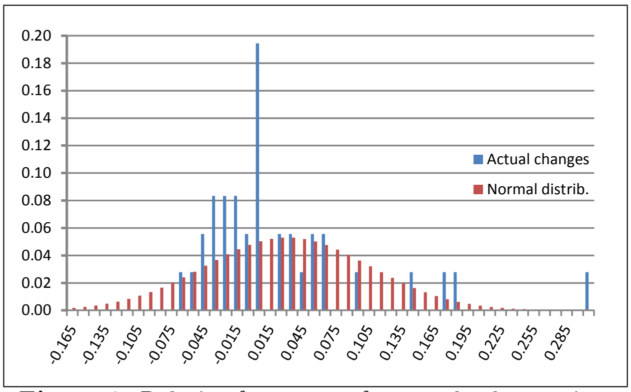

Figure 3. Relative frequency of quarterly changes in the price of one euro in ISK 2000Q1-2008Q4 and that of the normal distribution with the same mean and standard deviation.

Figure 3 shows that large increases in the price of the euro are more common than large decreases, and that the large increases exceed what is probable if the changes in the price are normally distributed. The most expensive mistakes in finance are frequently caused by underestimation of the probability of extreme events like the large depreciations of the krona in 2008.

3. Optimal currency hedging when utility is a function of the real value

The discussion in the previous section assumed that the investor's utility is a function of the nominal value of the portfolio. It is more realistic – especially in a country where there is significant inflation risk which is closely related to volatility in the exchange rate – to assume that the investor's utility is a function of the real value of the portfolio. In open economies there is positive correlation between the price of foreign currency and the rate of inflation which creates a hedge against the effects of changes in the exchange rates on the real value of foreign investments, diminishing the need for other hedges. In the extreme theoretical case, where all domestic goods are imported and the domestic price level changes as the foreign price level and the exchange rate, changes in the exchange rate do not affect the real value of the portfolio. In such a case buying some protection against changes in the nominal value of the portfolio does not make much sense.

If the domestic inflation in period t is denoted π_t^d, the real value of the portfolio by the end of period t can be found by dividing through Equation (5) by $1+\pi_t^d$ to obtain

$$\frac{W_{t+1}}{1+\pi_t^d} = (1+\rho_t)\,\frac{1+e_t}{1+\pi_t^d}W_t + h\cdot W_t\left(\frac{1+R_t^d}{\left(1+\pi_t^d\right)\left(1+R_t^f\right)} - \frac{1+e_t}{1+\pi_t^d}\right) \tag{11}$$

Simplifying by dropping terms which are the product of two small numbers as in Equation (5") above and subtracting c_t, the cost of the forward contract, appropriately, gives

$$\frac{W_{t+1}}{1+\pi_t^d} = \left(1+\rho_t+e_t-\pi_t^d\right)W_t + h\cdot W_t\left(R_t^d - R_t^f - e_t - c_t\right) \tag{11'}$$

Calculating expectations gives

$$\begin{aligned} E\left\{\frac{W_{t+1}}{1+\pi_t^d}\right\} &= \left[1+E\left\{\rho_t\right\}+(1-h)\,E\left\{e_t\right\}\right]W_t \\ &\quad -E\left\{\pi_t^d\right\}W_t + h\cdot W_t\left(R_t^d - R_t^f - c_t\right) \end{aligned} \tag{12}$$

and, assuming that ρ_t is independent of the other stochastic variables, e_t and π_t^d, gives

$$\begin{aligned} Var\left[\frac{W_{t+1}}{1+\pi_t^d}\right] &= \left[Var[\rho_t] + (1-h)^2\,Var[e_t]\right]W_t^2 \\ &\quad + \left[Var[\pi_t^d] - 2\,(1-h)\,Cov\left[e_t,\pi_t^d\right]\right]W_t^2 \end{aligned} \tag{13}$$

Substituting these expressions into the utility function in Equation (6), calculating first order condition w.r.t. h, and solving for the optimal value of the currency hedge gives

$$h^* = 1 + \frac{R_t^d - R_t^f - E\{e_t\} - c_t}{2\varphi Var\left(e_t\right)} - \frac{St.d\left(\pi_t^d\right)}{St.d\left(e_t\right)}Corr\left(\pi_t^d, e_t\right) \tag{14}$$

As the correlation between changes in the price of foreign currency and the rate of inflation, $Corr\left(\pi_t^d, e_t\right)$, is positive, optimal currency hedging is lower when the investor's utility is a function of the real value of the portfolio. Using data for the period 2000Q1-2008Q3 gives 0.0589 as the standard deviation of quarterly changes in the price of the euro, 0.0118 as the standard deviation of inflation in Iceland, and 0.686 as the contemporaneous correlation between these two variables. Equation (14) gives that the optimal currency hedge in this case is 13.7% lower when the investor's utility is a function of the real value of the foreign portfolio, than it is if utility is a function of the nominal value. This result will be discussed further below.

4. Stationary stochastic processes and random walk

Keating claims in SIC (2010) that hedging of currency risks is different for long term investors than it is for short term investors. It will be shown below that the validity of this claim depends on the characteristics of the stochastic process that the exchange rate follows. If the process is stationary and the aim is to hedge the currency risk in the long term, hedging of the currency risk in the present period is futile in most cases. But if the exchange rate follows the non-stationary random walk process, it is probably efficient to hedge the currency risk in each short period, also when the objective is to hedge the currency risk in the long term.

There are many studies that claim that nominal exchange rates and real exchange rates are not stationary, while others claim that they are. In a famous paper Meese and Rogoff (1983) showed that assuming that nominal exchange rates follow random walk, which means that it is optimal to use the most recent value on the exchange rate as a forecast for all future periods, gave better forecasts than other more complicated models. Lam e al. (2008) discuss the forecast performance of models of the nominal exchange rate while Taylor (2006) discusses the development of the consensus view on mean reversion in the real exchange rate and techniques used to analyse it, and claims that the consensus view today is that there is some mean reversion in the real exchange rates.

In the various versions of the Central Bank of Iceland's macroeconomic model the real exchange rate is estimated to be stationary (see Daníelsson e al. 2011) and it is probably widespread belief among observers and market participants that the real exchange rate of the Icelandic krona is stationary. If the real exchange rate is stationary it is probable that the nominal rate is also stationary, albeit possibly with a trend.

There are two characteristics of stationary stochastic processes that explain why it is probably not efficient to hedge the risk in the short term to insure against the long term risk. One is that reversion to the mean is built into a stationary process. If the exchange rate that follows a stationary process appreciates in the present period so that the value of the foreign investments decreases, the probability of depreciation has increased. This means that if the process is stationary the future changes in the exchange rate are negatively correlated with the change in the present period. Changes that are negatively correlated with changes in the present period form hedges against the risk in the present period, something that should be taken into account when calculating the optimal hedge. If, on the other hand, the exchange rate follows random walk, appreciation in the present period does not make a depreciation in subsequent periods more likely. For that reason there is no hedging implicit in expected future changes in the exchange rate and it is optimal to hedge the risk in the present period when insuring against the long term risk.

Formally this means that if $\widetilde{e}_t = \log(s_{t+1}) - \log(s_t) = \log(1 + e_t)$, the exchange rate at the start of the present period, t, is known, and $\log(s_t)$ is stationary then

$$Cov_t\left(\widetilde{e}_t, \widetilde{e}_{t+k-1}\right) = \left(b_{k-1} - b_{k-2}\right)\sigma^2, k = 2, 3, ... \tag{15}$$

where the coefficients (b_k) are coefficients in Wold representation of the process and $b_0 = 1$ (See Appendix A for details).

The covariance of the logarithmic change in the exchange rate in the current period $(\widetilde{e}_t)$ and the change in the exchange rate over $k-1$ subsequent periods is given by

$$Cov_t\left(\widetilde{e}_t, \log\left(s_{t+k}\right) - \log\left(s_{t+1}\right)\right) = Cov_t\left(\widetilde{e}_t, \sum_{j=1}^{k-1} \widetilde{e}_{t+j}\right) = b_{k-1}\sigma^2 - \sigma^2 \tag{16}$$

If the process is stationary the coefficients (b_{k-1}) decline to zero when k is large which means that for sufficiently large k correlation between changes in the exchange rate in the present period and changes in $k-1$ subsequent periods is negative. It now follows directly that the covariance between changes in the exchange rate in the present period, and changes in the exchange rate over k periods, from the beginning of period t, is given by

$$Cov_t\left(\widetilde{e}_t, \log\left(s_{t+k}\right) - \log\left(s_t\right)\right) = b_{k-1}\sigma^2 \tag{17}$$

which approaches zero when k is large.

Simple linear stochastic process that is often used in both theoretical and empirical work is given by

$$\log\left(s_t\right) = \log\left(s_{t-1}\right) + \alpha\left[\log\left(\overline{s}\right) - \log\left(s_{t-1}\right)\right] + \varepsilon_t \tag{18}$$

where $\overline{s}$ is the geometric mean of the process and α determines the degree of mean reversion or the "pull" towards the mean. If $0 < \alpha < 1$ the process is stationary, but if $\alpha = 0$ this process is random walk which is non-stationary and without any mean reversion. The coefficients in the Wold representation of this process are $b_j = (1-\alpha)^j$, $j = 0, 1, 2, ...$

It is convenient to use the process in Equation (18) to show how the change in the exchange rate in the present period is "corrected" in later periods so that it does not affect the long term change if the process is stationary. Figure 4 shows possible realisations of the stochastic process in Equation (18) when $\alpha = 0.26$ and $\sigma^2 = 0.061^2$. Assume that the value of the exchange rate is 1 at the start of period 1 and consider the possibilities of a 10% appreciation and a 10% depreciation in this period. It seems natural to hedge this risk, and if the funds are needed by the end of period 1 it certainly matters if the risk has been hedged or not.

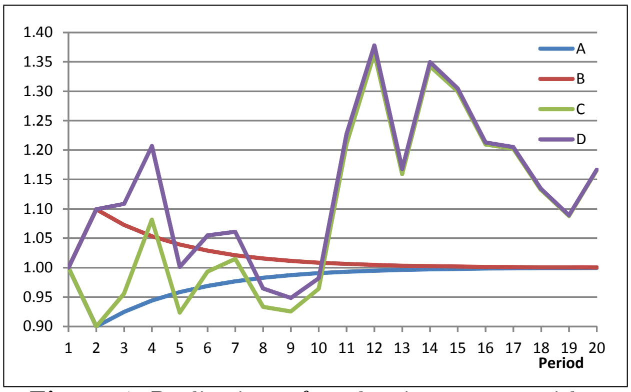

Figure 4. Realisations of stochastic processes with mean reversion.

But if the funds are not needed until period 20, or even period 10, the situation is quite different. Line A shows what happens in the case of a 10% decrease in the exchange rate index (s_t), which is equivalent to a 10% appreciation of the currency, followed by adjustment according to Equation (18), but without any further shocks, while line B shows what happens in the case of a 10% increase in the index followed by adjustments without further shocks. The Figure shows that quite soon the two processes converge. The same happens in the case of lines C and D where the former starts with a 10% decrease in the index in period 1 followed by shocks drawn from a normal distribution with variance 0.061^2, while the latter starts with a 10% increase followed by the same shocks in subsequent periods. Equation (18) where $\alpha > 0$ guarantees this kind of convergence, but the time required until the process converges depends on the value of α. If α is very low the convergence can be very slow, and if $\alpha = 0$ the process is no longer stationary but random walk, and there is no convergence at all. The difference established in period 1 continues unchanged in later periods.

The second characteristic of a stationary stochastic process that matters here is that the variance of long term change in the exchange rate is finite. This is part of the definition of a stationary process. Conditional on the information at the beginning of period t $Var_t(\log(s_{t+1})) = \sigma^2$ and $Var_t(\log(s_{t+n})) = \sigma^2 \sum_{j=0}^{n-1} b_j^2$. For the simple process in Equation (18) this means that $Var_t(\log(s_{t+n})) = \sigma^2 \sum_{j=0}^{n-1} (1-\alpha)^{2j} = \frac{1-(1-\alpha)^{2n}}{1-(1-\alpha)^2}\sigma^2$ if $\alpha > 0$, but $Var_t(\log(s_{t+n})) = n\sigma^2$ if $\alpha = 0$. Calculating the ratio between the long term risk and the risk in the present period (σ^2) gives n if $\alpha = 0$, and $\sum_{j=0}^{n-1} b_j^2$ or $\frac{1-(1-\alpha)^{2n}}{1-(1-\alpha)^2}$ if $\alpha > 0$. Figure 5 shows this ratio as a function of the number of periods (n) for some values on α.

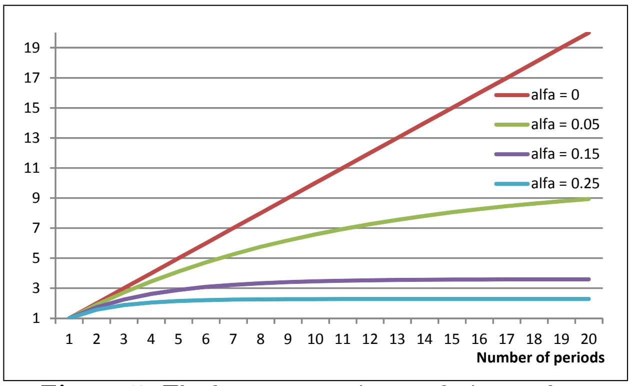

Figure 5. The long term variance relative to the variance in the present period

The figure shows that risk increases proportionally with the number of periods if $\alpha = 0$, but if $\alpha > 0$ the risk increases less than proportionally and even for low values on α the increase in the risk levels off after quite few periods. If the process exhibits mean reversion with $\alpha = 0.05$ the variance of the change over 20 quarters, or 5 years, is 9 times the variance of quarterly changes, and if $\alpha = 0.15$ the variance over 20 quarters is almost 4 times the variance of quarterly changes, while if $\alpha = 0.25$ the variance over 5 years is 2.29 times the variance of quarterly changes and does not increase above that even if the long period is extended beyond 20 quarters. If currency contracts are costly and the only contracts available are short term contracts with fixed cost per period, they may be too expensive for long term investors that need to hedge the currency risk over long periods of time. In the case of random walk the currency risk and the cost of hedging increases proportionally with the length of the period. In this case it seems probable that it is optimal to hedge in the long term in the same way as in the short period.

5. Currency hedging over the long period

A long term investor will consider the impact of the hedging of the currency risk in the present period on the value of the portfolio at a date in the future when the assets will be sold. In most cases the portfolio will be sold during some period of time, but here it will be assumed, for the sake of simplicity, that all assets will be sold at a given date in the future, say by the beginning of period $t+n$. At this time the value of the portfolio, assuming no additional investments or disinvestments, is obtained by using Equation (1) repeatedly. This gives that

$$W_{t+n} = W_t \prod_{j=1}^{n} \left(1 + \rho_{t+n-j}\right)\left(1 + e_{t+n-j}\right) \tag{19}$$

Using $\left(1 + R^d_{t+n-j}\right)$ as discount factors the present value at the start of period t of the value of the portfolio by the end of period $t+n-1$/beginning of period $t+n$ is

$$\frac{W_{t+n}}{\prod_{j=1}^{n}\left(1 + R^d_{t+n-j}\right)} = \frac{\prod_{j=1}^{n}\left(1 + \rho_{t+n-j}\right)\left(1 + e_{t+n-j}\right)}{\prod_{j=1}^{n}\left(1 + R^d_{t+n-j}\right)} W_t \tag{20}$$

As in Equation (5) in Section 2 with the cost of arranging the contract (c_t) included, the income from a currency hedge in period t is given by

$$h_t \cdot W_t \left(\frac{1 + R^d_t}{1 + R^f_t} - (1 + e_t) - c_t\right) \tag{21}$$

This expression gives the profit from a currency contract for $h \cdot W_t$ in period t, which is available by the end of the period. Its present value at the start of the period, when the decision on hedging is taken, is given by

$$\frac{h_t \cdot W_t}{1 + R^d_t}\left(\frac{1 + R^d_t}{1 + R^f_t} - (1 + e_t) - c_t\right) \tag{22}$$

The objective of the investor in the present period is to maximize the sum of the present value of the portfolio by the beginning of period $t+n$ and the present value of the profit from the currency hedge, given the risk involved. If the sum of the present values are given by

$$V_t = \frac{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)\left(1+e_{t+n-j}\right)}{\prod_{j=1}^{n}\left(1+R^d_{t+n-j}\right)} W_t + \frac{h_t \cdot W_t}{1+R^d_t}\left(\frac{1+R^d_t}{1+R^f_t} - (1+e_t) - c_t\right) \tag{23}$$

and the risk-preferences of the investor are given by the utility function

$$E_t\{U(V_t)\} = E_t\{V_t\} - \frac{\varphi}{W_t} Var_t(V_t) \tag{24}$$

the optimal currency hedge, calculated in the same way as above, is

$$h^*_t = \left(1+R^d_t\right)\frac{\frac{1+R^d_t}{1+R^f_t} - (1+E_t\{e_t\}) - c_t}{2\varphi Var_t(1+e_t)}$$
$$+\left(1+R^d_t\right)\frac{Cov_t\left[1+e_t, \frac{\prod_{j=1}^{n}(1+\rho_{t+n-j})(1+e_{t+n-j})}{\prod_{j=1}^{n}(1+R^d_{t+n-j})}\right]}{Var_t(1+e_t)} \tag{25}$$

The first term on the right hand side in Equation (25) is similar to the middle term in Equation (10). The differences are because of the linearization of $\frac{1+R^d_t}{1+R^f_t}$ and the use of discounting here. As discussed in Section 2 above, economic theory would predict that this term should be slightly negative, especially in the long term. Those that believe that it is positive can obviously act on this belief and invest in currency hedging for speculative profits. This case was discussed thoroughly in Section 2.

In the single period case where $n = 1$, assuming that the foreign rate of return is independent of the change in the exchanges rate, the second term in Equation (25) becomes:

$$\left(1+R^d_t\right)\frac{Cov_t\left[1+e_t, \frac{\prod_{j=1}^{1}(1+\rho_{t+n-j})(1+e_{t+n-j})}{\prod_{j=1}^{1}(1+R^d_{t+n-j})}\right]}{Var(1+e_t)}$$
$$= \frac{1+R^d_t}{1+R^d_t}(1+E_t\{\rho_t\})\frac{Var_t(1+e_t)}{Var_t(1+e_t)} = 1+E\{\rho_t\} \tag{26}$$

If the nominator in the middle term in Equation (10) is zero the efficient currency hedge is $h^* = 1$ but Equation (25) gives $1 + E\{\rho_t\}$ in this case. The difference, $E\{\rho_t\}$, is explained by the approximations used in deriving Equation (10). If there is no cost involved in hedging the currency risk and the other stochastic variable in the model, the foreign return, is independent of the currency risk, the risk averse investor will hedge all of the currency risk, both on the principal and on the expected return.

It seems realistic to assume that in the case of small monetary area like Iceland the risk in the foreign return is indpendent of the risk in the exchange rate. R^d_t is given, but it is realistic to assume that future domestic short term rates are stochastic which makes it necessary to make some assumptions about them. If either: 1) $\prod_{j=1}^{n}\left(1+R_{t+n-j}^{d}\right)$ is stochastically independent of both $1+e_t$, and $\prod_{j=1}^{n}(1+e_{t+n-j})^2$, or 2) if $\prod_{j=1}^{n}\left(1+R_{t+n-j}^{d}\right)$ is independent of $1+e_t$, and $\left(1+R_{t+n-j}^{d}\right)$, and $(1+e_{t+n-j})\,, j=1,2,..,n$ are log-normally distributed, the covariance term in the second term in Equation (25) can be written as (see Appendix B for some details)

$$Cov_t\left[1+e_t, \frac{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)\left(1+e_{t+n-j}\right)}{\prod_{j=1}^{n}\left(1+R_{t+n-j}^{d}\right)}\right]=$$

$$=\frac{E_t\left\{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)\right\}E_t\left\{1+e_t\right\}\prod_{j=1}^{n}E_t\left\{1+e_{t+n-j}\right\}}{\prod_{j=1}^{n}E_t\left\{1+R_{t+n-j}^{d}\right\}}\left[\exp\left[b_{n-1}\sigma^2\right]-1\right] \tag{27}$$

where b_{n-1} is a coefficient in the Wold representation of the exchange rate discussed in the previous section.

Substituting this into Equation (25) above gives

$$h_t^{*}=\left(1+R_t^{d}\right)\frac{\frac{1+R_t^{d}}{1+R_t^{f}}-\left(1+E_t\{e_t\}\right)-c_t}{2\varphi Var_t\left(1+e_t\right)}$$

$$+\frac{E_t\left\{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)\right\}\prod_{j=1}^{n-1}E_t\left\{1+e_{t+n-j}\right\}}{E_t\left\{\prod_{j=1}^{n-1}\left(1+R_{t+n-j}^{d}\right)\right\}}\cdot\frac{\exp\left[b_{n-1}\sigma^2\right]-1}{\exp\left[\sigma^2\right]-1} \tag{25'}$$

To analyse the second term on the right hand side of Equation (25') it is convenient see it as composed of two factors separated by the multiplication sign. It seems a reasonable first approximation to expect that the first factor increases/decreases by the factor $\frac{E\left\{1+\rho_{t+n-1}\right\}E\left\{1+e_{t+n-1}\right\}}{E\left\{1+R_{t+n-1}^{d}\right\}}$ when the length of the time until the assets will be sold is increased by one period. This factor is smaller if the domestic interest rates are high compared to foreign returns and expected changes in the exchange rate. Using that $\exp[a]\approx 1+a$ if a is small gives that the second factor can be approximated by b_{n-1} which approaches zero when n is large. In the case where the exchange rate follows a stochastic process with

[2] It seems reasonable to assume that the domestic interest rates are sufficiently forward looking so that they are independent of past changes in the exchange rate but they might be correlated, probably positively correlated, with present or future changes in the exchange rate. There are periods where the correlation between short term interest rates in Iceland and changes in the exchange rate are slightly positive but over the period from April 2001 when the new regime of inflation targeting and floating exchange rate had been introduced to December 2007 when the financial system was set on downhill path that lead to bankruptcy of the big banks, the correlation was 0.041. That seems to justify the use here of the assumption that the domestic rates are independent of the changes in the exchange rate.

mean reversion defined in Equation (18) in the previous section $b_{n-1} = (1-\alpha)^{n-1}$ which decreases by the factor $1-\alpha$ when the time is increased by one period. Except for very low values on α, $(1-\alpha)$ can be expected to be sufficiently small to outweigh possible increases in the first factor so that the product in the second term in Equation (25') decreases and eventually approach zero when the number of periods is increased.

If the exchange rate follows random walk $b_{n-1} = 1$ and $\frac{\exp[b_{n-1}\sigma^2]-1}{\exp[\sigma^2]-1} = 1$ in all periods. In this case Equation (25') gives that the efficient hedging for long term investors depends on the value of first factor in the second term. If it is close to unity, which seems a reasonable first approximation, the efficient hedging of the currency risk in the present period is the same for long term investors as it is for short term investors.

The formula in Equation (23) uses the present value where the domestic short term rate is used for discounting. This rate is nominal. If it is decomposed into the real rate, r_t^d, and inflation then

$$1 + R_{t+n-j}^d = \left(1 + \pi_{t+n-j}^d\right)\left(1 + r_{t+n-j}^d\right) \tag{28}$$

If the rate of inflation depends positively on the change in the exchange rate the following formulae gives a reasonable and simple description of the pass through (note that possible errors are ignored)

$$1 + \pi_{t+n-j}^d = (1 + e_{t+n-j})^{\beta}(1 + w_{t+n-j})^{1-\beta} \tag{29}$$

where w_{t+n-j} is the change in an index of other variables that affect the rate of inflation.

Substituting from Equations (28) and (29) into Equation (23) gives after simplifications

$$V_t = \frac{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)(1+e_{t+n-j})^{1-\beta}}{\prod_{j=1}^{n}\left(1+r_{t+n-j}^d\right)(1+w_{t+n-j})^{1-\beta}} W_t + \frac{h_t \cdot W_t}{1+R_t^d}\left(\frac{1+R_t^d}{1+R_t^f} - (1+e_t) - c_t\right) \tag{23'}$$

Solving for the optimal currency hedging in this case, using same kind of assumptions on distributions of stochastic variables, including that $\prod_{j=1}^{n-1}\left(1+r_{t+n-j}^d\right)$ and $\prod_{j=1}^{n-1}(1+w_{t+n-j})$ are independent of other variables, and log-normally distributed, gives that

$$h_t^* = \left(1+R_t^d\right)\frac{\frac{1+R_t^d}{1+R_t^f} - (1+E\{e_t\}) - c_t}{2\varphi Var(1+e_t)}$$

$$+\frac{E\left\{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)\right\}\prod_{j=1}^{n-1}E\left\{(1+e_{t+n-j})^{1-\beta}\right\}}{E\left\{\prod_{j=1}^{n-1}\left(1+r_{t+n-j}^d\right)(1+w_{t+n-j})^{1-\beta}\right\}}\frac{\exp\left[(1-\beta)\,b_{n-1}\sigma^2\right]-1}{\exp\left[\sigma^2\right]-1} \tag{25''}$$

If the first factor in the second term in Equations (25”) is unity, the second term can be approximated by $(1-\beta)\, b_{n-1} \approx \frac{\exp\left[(1-\beta)b_{n-1}\sigma^2\right]-1}{\exp[\sigma^2]-1}$.

It was assumed above that the first term in Equations (25’) and (25”) was not positive. If the UIP is valid the nominator in this term should be zero or slightly negative which means that the optimal hedging is lower than indicated by the second term and possibly negative. As in the one period case, which was discussed in Section 2 above, speculative investments in forward currency contracts can be very tempting if the first term in Equations (25’) and (25”) is believed to be positive and possibly large. The optimal purchase of currency hedge is though lower if the funds are to be used in distant future rather than by the end of the present short period as all benefits from the investments are in the form of returns on the investment in forward contracts for the currency (the first term in the equations), but the hedging of the currency risk does not contribute any benefits in the form of decreasing the variance in the value of the portfolio (the second term is zero).

6. Estimation of a simple model for Iceland

Figure 6 shows indices for the nominal and real exchange rates of the Icelandic krona from 2000Q1 to 2013Q4. In 2000 the krona was officially on a fixed exchange rate but that regime was breaking down at the time, almost a decade after it broke down in Europe. Floating exchange rate, together with inflation targeting, was adopted in March 2001.

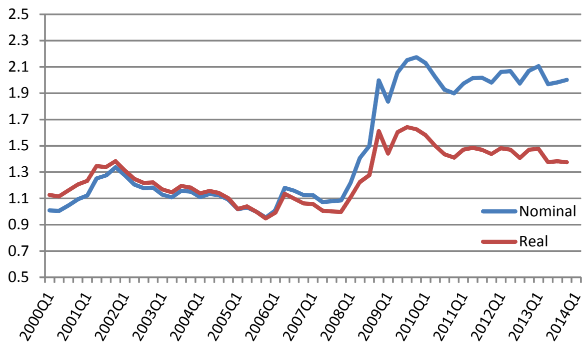

Figure 6. Indices of nominal and real exchange rate of the Icelandic krona (the price of foreign currency). Source: Central Bank of Iceland.

Formal tests for unit roots in the series in Figure 6 show that the null hypothesis of a unit root cannot be rejected, except if the shift in 2008 is somehow subtracted (e.g. by using estimation of the shift in Equation (30) below). In a way that is to be expected, especially for such short time series, as these tests are known to have very low power. Looking at Figure 6 indicates that there may be some mean reversion in the nominal rate, but also a significant shift in 2008, a shift that is unlikely to be reversed. There is also a clear shift in the real rate, but the shift is smaller and it can be argued that it will be reversed. It therefore seems more realistic in the case of the Icelandic economy to start with the assumption that it is the real exchange rate that is stationary rather than the nominal rate.

Estimating the simple stationary linear process in Equation (18) for the real exchange rate (z_t) with a shift dummy ($S083$) taking the value of 1 from 2008Q3 but zero before, using data from 2000Q1 to 2012Q4, gives

$$\log\left(z_t\right) = \underset{(0.0901)}{0.742} \log\left(z_{t-1}\right) + \underset{(0.0290)}{0.077}\, S083 + \underset{(0.0154)}{0.033} \tag{30}$$

where standard deviations of the estimates are in the parenthesis. The standard error of the regression is $\sigma_z = 0.057$. The coefficient 0.742 means that $\alpha = 0.258$. The mean of the process is $0.033/0.258 = 0.128$ until 2008Q2 but 0.426 after that.

If the shift dummy is excluded from the equation the value of $\alpha = 0.064$, the constant is not significant and the standard error of the regression is $\sigma_z = 0.061$.

Estimating the contemporaneous pass through equation

$$\log\left(1 + \pi_t^d\right) = \beta \log\left(1 + e_t\right) + \omega + \varepsilon_t^{\pi d} \tag{31}$$

gives $\beta = 0.118$ and $\omega = 0.012$. In the previous section it was explained that if there is contemporaneous pass through of β, the efficient level of currency hedging decreases by the same value when the investor considers the real value of the portfolio rather than the nominal value. The value here, 11.8%, is similar to the 13.7% obtained from Equation (14) in Section 3.

Adding lagged changes in the exchange rate improves the estimated equation which is now

$$\log\left(1 + \pi_t^d\right) = \underset{(0.01433)}{0.1092} \log\left(1 + e_t\right) + \underset{(0.01433)}{0.0845} \log\left(1 + e_{t-1}\right) + \underset{(0.00096)}{0.01148} \tag{32}$$

$R^2 = 0.68$, and the standard error of the regression is $\sigma_{\pi^d} = 0.0067$.

Including Equation (32) in a model of long term investments makes the formula for the optimal currency hedge a lot more complicated than the one in Section 5. A much simpler method which should give approximately correct results for hedging the long term risk would be to use the value of the long term pass through in Equation (25") in Section 5. The sum of the pass through coefficients in Equation (32) are $0.1092 + 0.0845 \approx 0.2$. Given that the share of export and imports in the GDP in Iceland is roughly 50% it is reasonable to expect that the implicit hedging though the pass through from changes in the exchange rate into inflation in Iceland for a long term investor who's utility is a function of the real value of the portfolio is even higher than 0.2. Pétursson (2008), allowing for the effects of changes in the exchange rate on inflation during two years, obtained a value for pass through in Iceland of 0.43. This value is among the highest in the group of mostly OECD countries studied in this paper. Israel is at the top with a pass through of 0.79 while UK and the USA are at the bottom with almost no pass through, 0.04 and 0.02 respectively.

It is shown in Appendix C that if the real exchange rate follows a stochastic process with mean reversion as in Equation (30), the nominal rate follows a process with the same mean reversion towards a trend which is determined by the difference in the rate of inflation at home and abroad. The trend may be partly stochastic which means that it contains a random walk term. This would e.g. happen if Equation (31) would be included in a model with Equation (30). If the foreign rate of inflation is modelled as the sum of the mean and an error term it would also contribute a random walk process to the formula for the nominal exchange rate. Using the period 2000Q1-2012Q4 gives the standard deviation for the foreign rate of inflation as $\sigma_{\pi^f} = 0.0047$. In this case the covariance between the change in the logarithm of the nominal exchange rate in the present period and the change in the next k periods is given by

$$Cov_t\left(\widetilde{e}_t, \log\left(s_{t+k}\right) - \log\left(s_t\right)\right) = \frac{\left(1-\alpha\right)^{k-1}\sigma_z^2 + \sigma_{\pi^d}^2 + \sigma_{\pi^f}^2}{\left(1-\beta\right)^2} \tag{33}$$

and the ratio between the covariance in the deviation of the nominal rate from the trend in the long term, given by Equation (33) when k is large, and the variance in the present period, given by Equation (33) when $k = 1$, becomes 0.024 if $\alpha = 0.258$, $\sigma_z = 0.06$, $\sigma_{\pi^d} = 0.0067$, $\sigma_{\pi^f} = 0.0047$, and $\beta = 0.2$. This number is very close to zero which was obtained in the previous section assuming that the nominal exchange rate follows a stationary stochastic process and indicates that efficient currency hedging in the short period for the Icelandic long term investors is close to zero. Adding the hedging from the considerable pass through from changes in the exchange rate into inflation in Iceland makes it improbable that the Icelandic pension funds have been able to reduce the long term currency risk in their foreign investments through purchasing short term forward contracts.

7. Discussion

This paper discusses optimal hedging of currency risks under different circumstances. It was shown that if hedging is costless and foreign return is independent of changes in the exchange rate, it is optimal to hedge all of the currency risk if the investor's utility is a function of the nominal value of the portfolio by the end of the short period, but less - possibly much less - if part of the currency risk is hedged through the impact of changes in the exchange rate on inflation, and the investor's utility is a function of the real value of the portfolio. The optimal hedging ratio declines with the number of periods until the money will be used and eventually becomes zero in most cases where the exchange rate follows a stationary stochastic process. If, on the other, the exchange rate follows random walk, it is optimal in most cases to hedge the currency risk in the short period.

If there is expected profit from hedging the currency, because the interest rate differential exceeds expected depreciation of the currency, investing in currency hedges becomes a tempting speculative investment opportunity and it may be optimal for risk averse investors to invest in these instruments for the return, even if it is of no use for hedging the relevant currency risks. The belief that the interest rate differential between Iceland and other countries exceeded (and will exceed?) the expected depreciation of the krona seems to have been the primary motivation for Icelandic pension funds' purchases of forward currency contracts. This belief (and bet) is exactly the same belief which motivated the carry trade where foreign institutions issued bonds denominated in Icelandic kronor with high rates of interest, the so called glacier bonds that were issued from the autumn of 2005 until 2008, and sold to foreign investors. The buyers of glacier bonds would usually not hedge the currency risk, but the issuers, mostly financial institutions with high credit rating, frequently did hedge their exposure to the krona by taking the proceeds from the sale of glacier bonds to Iceland, exchanged the sum into kronor and lent it mostly to Icelandic banks. In Section 2 above it was shown that there are recent periods when the difference between short term rates in Iceland and foreign short term rates exceeded the change in the price of foreign currency making speculative investment in currency hedges very profitable and also periods when this did not hold. It adds to the risk of these investments that the distribution of changes in the exchange rate of the krona is skewed so that large depreciations are more probable than large appreciations.

Burnside (2013) finds that the risk of large depreciations explains that the interest rates in New Zealand are on the average higher than foreign interest rates. This sounds a bit like the Icelandic situation. Both countries were targets of large volumes of carry trade during its hay days in 2005-2007. If interest rates in one country can be permanently higher than in other countries for some such reason, and in spite of free movements of capital, it is to be expected that other returns are also higher making it optimal for domestic investors in the high return country to invest more at home.

It was noted in the introduction that this paper discusses hedging of foreign investments of an investor that does not have any domestic assets. This is appropriate for analysing the mechanisms determining the optimal currency hedge for foreign investments, but it is, of course, incomplete as a description of the situation of a domestic investor that allocates some funds to foreign investments to enlarge the set of investment opportunities and to decrease volatility in the portfolio. The relative returns at home and abroad, and the correlation between them, should determine the optimal allocation of funds to foreign investments. In a proper setting currency hedging should be a part of a strategy which includes the decision on the share of foreign investments in the portfolio. The differences in returns between the domestic and the foreign markets would then not only affect the level of currency hedging for the foreign investments, but also the level of the foreign investments. The currency risk would also get the proper risk weight in relation to the total portfolio. This would though not change the main result in this paper that a long term investor cannot reduce long term currency risk in foreign investments by buying short term forward contracts if the nominal exchange rate of the currency follows a stationary process with not negligible mean reversion.

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Appendix A

Wold's representation theorem (see e.g. Hamilton, 1994, p. 109) states that a stationary stochastic process can be expressed as a weighted sum of uncorrelated stochastic variables. This means that if the logarithm of the exchange rate s_t, at the start of period t, follows a stationary stochastic process it is possible to write

$$\log(s_t) = \sum_{j=0}^{\infty} b_j \varepsilon_{t-j} + \eta_t \tag{A.1}$$

where ε_{t-j} are uncorrelated stochastic variables with zero mean and variance σ^2, and η_t is the deterministic component of the process, assumed uncorrelated with the stochastic part. The coefficients, b_j, are such that $\sum\limits_{j=0}^{\infty} b_j^2 < \infty$. It is customary to assume that $b_0 = 1$.

If the realisation of ε_{t-j} is written with a hat as $\widehat{\varepsilon}_{t-j}$, then if s_t is known it can be written as $\log(\widehat{s}_t) = \sum\limits_{j=0}^{\infty} b_j \widehat{\varepsilon}_{t-j} + \eta_t$. Conditional on the information available at the start of period t the exchange rate at the start of period $t+1$ can be written as $\log(s_{t+1}) = b_0 \varepsilon_{t+1} + \sum\limits_{j=1}^{\infty} b_j \widehat{\varepsilon}_{t+1-j} + \eta_{t+1}$, and conditional on the same information the exchange rate at the start of period $t+k$ can be written as $\log(s_{t+k}) = \sum\limits_{j=0}^{k-1} b_j \varepsilon_{t+k-j} + \sum\limits_{j=k}^{\infty} b_j \widehat{\varepsilon}_{t+k-j} + \eta_{t+k}$. It is now easy to see that, conditional on the information available at the start of period t, $Cov_t(\log(s_{t+1}), \log(s_{t+k})) = b_0 b_{k-1} \sigma^2 = b_{k-1}\sigma^2 \quad \text{if } b_0 = 1$.

Let the logarithmic change in the exchange rate in period t be $\widetilde{e}_t$, which means that $\widetilde{e}_t = \log(s_{t+1}) - \log(s_t) = \log(1 + e_t)$. Conditional on the information available at the start of period t we have that

$$Cov_t\left(\widetilde{e}_t, \widetilde{e}_{t+k-1}\right) = Cov_t\left(\log\left(s_{t+1}\right) - \log\left(\widehat{s}_t\right), \log\left(s_{t+k}\right) - \log\left(s_{t+k-1}\right)\right)$$

$$= b_{k-1}\sigma^2 - b_{k-2}\sigma^2 = \left(b_{k-1} - b_{k-2}\right)\sigma^2, k = 2, 3, ... \tag{A.2}$$

It also follows that the covariance of the change in the exchange rate in the present period and the change from the end of the present period until the start of period $t + k$ is

$$Cov_t\left(\widetilde{e}_t, \log\left(s_{t+k}\right) - \log\left(s_{t+1}\right)\right) = Cov_t\left(\widetilde{e}_t, \sum_{j=1}^{k-1} \widetilde{e}_{t+j}\right)$$

$$= \sum_{j=1}^{k-1} Cov_t\left(\widetilde{e}_t, \widetilde{e}_{t+j}\right) = \sum_{j=1}^{k-1}\left(b_j - b_{j-1}\right)\sigma^2 = b_{k-1}\sigma^2 - \sigma^2 \tag{A.3}$$

As b_{k-1} approaches zero when the stochastic process is stationary, and k is large, this means that correlation between the change in the exchange rate in the present period, and the change in the subsequent long period, is negative. This means that it is probable that the risk in the exchange rate in the present period will be mitigated through the changes in the exchange rate in later periods reducing the efficient hedging of currency risk in the present period for the long term investor.

The covariance between the change in the logarithm of the exchange rate in the present period and the change from the start of the present period until the start of period $t + k$ is then

$$Cov_t\left(\widetilde{e}_t, \log\left(s_{t+k}\right) - \log\left(s_t\right)\right) = Cov_t\left(\widetilde{e}_t, \sum_{j=0}^{k-1} \widetilde{e}_{t+j}\right)$$

$$= b_{k-1}\sigma^2 - \sigma^2 + \sigma^2 = b_{k-1}\sigma^2 \tag{A.4}$$

This means that, if the exchange rate follows a stationary stochastic process, the correlation between the currency risk in the present period and the long term currency risk that matters for long term investors approaches zero when k is large, making it irrelevant for long term investors to hedge against the short term risk.

Simple linear stochastic process that is often used in both theoretical and empirical work is given by

$$\log\left(s_t\right) = \log\left(s_{t-1}\right) + \alpha\left[\log\left(\overline{s}\right) - \log\left(s_{t-1}\right)\right] + \varepsilon_t \tag{A.5}$$

where $\overline{s}$ is the geometric mean of the process and α determines the degree of mean reversion or the "pull" towards the mean. The Wold representation for this process is

$$\log\left(s_t\right) = \sum_{j=0}^{\infty}\left(1 - \alpha\right)^j \varepsilon_{t-j} + \overline{s} \tag{A.6}$$

with the coefficients $b_j = \left(1 - \alpha\right)^j, j = 0, 1, 2, ...$

Appendix B

If X, Y and Z are stochastic variables and Z is independent of the other two then

$$Cov\left(X,YZ\right)=E\left\{XYZ\right\}-E\left\{X\right\}E\left\{YZ\right\}=E\left\{XY\right\}E\left\{Z\right\}-E\left\{X\right\}E\left\{Y\right\}E\left\{Z\right\}$$

$$=E\left\{Z\right\}\left[E\left\{XY\right\}-E\left\{X\right\}E\left\{Y\right\}\right]=E\left\{Z\right\}Cov\left(X,Y\right)$$

It follows that

$$Cov_t\left[1+e_t,\frac{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)\left(1+e_{t+n-j}\right)}{\prod_{j=1}^{n}\left(1+R^d_{t+n-j}\right)}\right]$$

$$=E_t\left\{\prod_{j=1}^{n}\left(1+\rho_{t+n-j}\right)\right\}Cov_t\left[1+e_t,\frac{\prod_{j=1}^{n}\left(1+e_{t+n-j}\right)}{\prod_{j=1}^{n}\left(1+R^d_{t+n-j}\right)}\right] \tag{B.1}$$

It is known from statistical theory that if X and Y are log-normally distributed then

$$Cov\left(X,Y\right)=E\left\{X\right\}E\left\{Y\right\}\left[\exp\left[Cov\left(\log\left(X\right),\log\left(Y\right)\right)\right]-1\right]$$

This means that if $\left(1+R^d_{t+n-j}\right)$ and $\left(1+e_{t+n-j}\right), j=1,2,...,n$ are log-normally distributed then

$$Cov_t\left[1+e_t,\frac{\prod_{j=1}^{n}\left(1+e_{t+n-j}\right)}{\prod_{j=1}^{n}\left(1+R^d_{t+n-j}\right)}\right]=\frac{E_t\left\{1+e_t\right\}\prod_{j=1}^{n}E_t\left\{1+e_{t+n-j}\right\}}{\prod_{j=1}^{n}E_t\left\{1+R^d_{t+n-j}\right\}}$$

$$\cdot\left[\exp\left[Cov_t\left(\log\left(1+e_t\right),\sum_{j=1}^{n}\log\left(1+e_{t+n-j}\right)-\log\left(1+R^d_{t+n-j}\right)\right)\right]-1\right]$$

$$=\frac{E_t\left\{1+e_t\right\}\prod_{j=1}^{n}E_t\left\{1+e_{t+n-j}\right\}}{\prod_{j=1}^{n}E_t\left\{1+R^d_{t+n-j}\right\}}\left[\exp\left[b_{n-1}\sigma^2\right]-1\right] \tag{B.2}$$

where the last line uses Equation (17) in Section 4 and that $\log(1+e_t)$ and $\log\left(1+R^d_{t+n-j}\right), j = 1, 2, ..., n$, are independent.

Appendix C

It was explained in Appendix A that if the real exchange rate follows a stochastic process with mean reversion like

$$\log(z_t) = \log(z_{t-1}) + \alpha(\log(\overline{z}) - \log(z_{t-1})) + \varepsilon^z_t \tag{C.1}$$

it has the Wold representation

$$\log(z_t) = \log(\overline{z}) + \sum_{j=0}^{\infty}(1-\alpha)^j \varepsilon^z_{t-j} \tag{C.2}$$

As the definition of the real exchange rate gives that

$$\log(z_t) = \log(s_t) - \log\left(P^d_t\right) + \log\left(P^f_t\right) \tag{C.3}$$

where P^d_t is the domestic price level, P^f_t is the foreign price level, and s_t is the nominal exchange rate.

Substituting from Equation (C.3) into (C.2) gives

$$\log(s_t) = \log\left(P^d_t\right) - \log\left(P^f_t\right) + \log(\overline{z}) + \sum_{j=0}^{\infty}(1-\alpha)^j \varepsilon^z_{t-j} \tag{C.4}$$

which shows that if $\log(z_t)$ follows a stochastic process with mean reversion $\log(s_t)$ also follows a stochastic process with the same mean reversion towards a trend determined by the difference between the trends in the two price levels.

Using a pass through equation to substitute for $\log\left(P^d_t\right)$ gives an equation that can be used to calculate the covariance needed for estimating the optimal currency hedge in Section 5. These calculations show that the covariance of the change in the present short period and the very long term do not converge all the way to zero. The reason is that including a pass though equation like Equation (31) in Section 6 makes $\log\left(P^d_t\right)$ in Equation (C.4) non-stationary. Assuming that the foreign rate of inflation is the sum of a deterministic part and a stochastic error term implies also that $\log\left(P^f_t\right)$ is non-stationary. Data for Iceland show that the variance of the error term in the pass through Equation (32) and the variance of the error term in the foreign inflation are much lower than the variance in the real exchange rate, and therefore the ratio of the covariance and the variance, the last factor in the last term on the right-hand side of Equations (25') and (25"), does converge to a very low number even if not quite to zero.