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QMM: A steady state version

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Abstract

This paper studies long-run characteristics of the quarterly macro model (QMM) used at the Central Bank of Iceland; it studies if there exists a balanced growth path that QMM will replicate and if it will converge to this path. It concludes that there is no such path and therefore the model does not converge to it. The paper then studies which adjustments to QMM are required to produce a model for which there exists a realistic balanced growth path. The new model is derived with minimum changes to specific equations in QMM and should therefore retain its key dynamic properties. The paper checks this by comparing impulse-responses from the new balanced growth compatible model to those from QMM. Finally, the paper studies some important variables in the Icelandic economy: capital output ratio, share of wage cost, real rate of interest and equilibrium real exchange rate and their calibration for QMM. It also discusses calibration of long-run values for the exogenous variables in the model and uses the balanced growth compatible model together with these long-run values of the exogenous variables to estimate equilibrium values for endogenous variables in QMM.

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1. Introduction

The Quarterly Macroeconomic Model (QMM) has been used for macroeconomic forecasting and monetary policy analysis at the Central Bank of Iceland since 2006. Since then some changes have been made to the model. Presently, the bank uses version 2.0 of the model, which was introduced in 2008. In this version, forward looking equations were included for the first time.[1] The equations for consumer prices (CPI), long-run rate of interest, real exchange rate and the monetary policy rule contain variables dated in future periods on the right hand side. The basic structure of the model is the same as in earlier versions but the estimations of the equations have been updated using data until 2006Q4 instead of 2004Q4. In some cases re-estimation of an equation did require some changes in its structure.

When QMM was constructed, efforts were made to ensure that equations in the model had some intuitive long-run properties. The production function is homogenous of degree one in capital and technically augmented labour and if seasonal factors and dummies are ignored most price equations[2] are homogenous of degree one in the level of prices so that they allow all paths where all relative prices remain constant. Care was taken so that those equations that are based on the error correction methodology, where the errors from an estimated long-run relationship are included in the equation explaining the short-run behaviour, were formulated so that they are compatible with a balanced growth path where all real variables change at some given rate determined by technical progress and population growth and where all prices change at some given rate, leaving relative prices unchanged. On the other hand, no efforts were made to ensure that the model as a whole was consistent with a balanced growth path.

Until quite recently, most central banks used macro models similar to QMM. Today many central bank have moved to DSGE models for macroeconomic forecasting and policy analysis. These models are based on the assumption that there exists an equilibrium growth path which the model gravitates towards in the absence of shocks. The final formulation of the theoretical model is in terms of deviations from this balanced growth path and much of the work on the statistical estimations of the models is allocated to efforts to locate this growth path. The main purpose of the present paper is to construct and calibrate such an equilibrium or balanced growth path for QMM.

QMM is also different from DSGE models in that it is not microfounded and therefore exposed to the Lucas critique. The handbook for version 2.0 of QMM notes that "(t)he degree of empirical coherence is therefore given some precedence

[1]See Daníelsson et al. (2009). The model described in this paper is derived from QMM. The handbook for QMM contains detailed explanation of the equations in the model and their estimations.

[2]The equation for the unit labour cost is an exception here. It was estimated with a constant and a trend. In forecasting this trend is not used but the value of the term containing the trend is fixed. Cf. the discussion below about the trend in the share of wage cost in Iceland.

The CPI equation is homogenous of degree one in prices and the inflation target.

over the degree of full theoretical coherence in Pagan's (2003) terms."[3] The Central Bank of Iceland is presently working on the construction of a DSGE model for the Icelandic economy which will be used along with QMM.

This paper is organised as follows: Section 2 presents long-run simulations with QMM that start from a recent quarter and assume some realistic paths for the exogenous variables. It is shown that these simulations do not converge in the long-run because the level of total expenditure is unsustainable. Given the fact that the Icelandic economy has been running an external deficit and sometimes a large deficit it shouldn't come as a surprise that a statistically estimated model simulates a path where there is an unsustainable external deficit and debt explodes. In most cases the net financial wealth of households also decreases (net debt increases) in relation to disposable income. The simulations show that this ratio converges towards levels which are far below the present rather low level (i.e. high ratio of net debt to disposable income).

Section 3 dicusses more formally the conditions for the existence of a balanced growth path for QMM. Section 3.1 describes the system of equations that constitutes necessary conditions for the existence of a balanced growth path for the model. It states the conclusions from efforts to solve this system of equations for QMM that in most cases, i.e. for most realistic assumptions concerning values for the exogenous variables, lead to the conclusion that there does not exist a balanced growth path for the model. For this reason a new model, QMM-BG, is constructed, a model which is as close to the original QMM as possible, but where some equations have been adjusted so that there exists a solution for the balanced growth path and where the debt levels of households and the external debt of the Icelandic economy are sustainable and realistic.

Section 4 discusses some important relationships in QMM and their role for determining the balanced growth path. Section 5 points out subsets of equations that can be used to solve for the balanced growth path and uses this result to give some intuition for why QMM does not converge to such a path. It also discusses how some equations in QMM should be altered to construct QMM-BG. The natural approach in constructing QMM-BG is to rescale the relevant equations in QMM by adding a constant. This should mostly affect the long-run properties of the model but leave the short and medium term dynamic properties intact.

The balanced growth solution giving QMM-BG depends on the assumptions made concerning the long-run paths of the exogenous variables. Section 6 discusses calibration of the exogenous variables as well as the calibration of the wage share, the capital-output ratio and the equilibrium risk free real interest rate. It is argued that the equilibrium solutions to the calibrated model provides estimates of long-run equilibrium values for the real exchange rate and the real wage.

Section 7 studies the sensitivity of important equilibrium values to some changes in the values of the exogenous variables. Section 8 compares impulse-responses from QMM-BG to those that can be obtained from a backward version of QMM by comparing a baseline forecast and a forecast with a specified change in some variable. As

[3] Daníelsson et al. (2009), p. 15.

one would expect there are some small differences similar to those that one would also get by measuring the implulse-responses with the backward version of QMM but starting from a different point.

Impulse-responses from the forward looking verions of QMM have also been compared to impulse-responses from the forward looking version of QMM-BG. The preliminary results are that the impulse-responses obtained when a shock is applied to the forward looking version of QMM-BG are much smaller than those obtained by comparing two runs of the forward version of QMM. These matters will be studied further and in another paper.

Section 9 concludes.

2. Long-run simulations with QMM

Figure 2.1 below shows results from a long-run simulation with the backward-looking version of QMM 2.0. The figures start in 1995Q1 and show historical data until 2008Q2 and then simulated data for the period 2008Q3-2200Q1. The figures show that the paths do not converge towards some balanced growth path. Instead the share of consumption (C) and real disposable income ($RHPI$) in GDP increases, leading to ever increasing ratio of current account deficit (BAL) to nominal GDP ($GDPN$) and ratio of net foreign debt (NFA) to nominal GDP.[4]

The ratio of households' net financial debt (NFW) to nominal disposable income ($PC \times RHPI$) increases quite fast from roughly -3.5 in 2008Q2 to a value which is roughly -10 and remains at this very low level.

It will be argued below that if the public consumption is 23.6% of GDP, as it has been on average during recent years, and if investments are 24.3% of GDP, which follows from the historical capital-output ratio and the estimated rate of depreciation, then a share of private consumption of almost 60%, which has been the case in recent years, leads to an unsustainable external deficit. Figure 2.1 shows that in long-run simulations with QMM, private consumption rises well above 60% of GDP.

Section 8 shows that if that if the backward version of QMM-BG is to return to a balanced growth path after being subjected to a shock, it is necessary to alter the equation for the real exchange rate from the backward version of QMM 2.0. This is done by adding a term representing increasing risk premium on foreign lending to Iceland when the level of indebtedness increases to the equation in QMM-BG.

If this term is added to the real exchange rate equation in the backward version of QMM, the real exchange rate declines further than in the case shown in Figure 2.1 leading to smaller current account deficits and slower downward slide in the ratio of

[4] Variables not explained elsewhere are: C=Private consumption; G=Public consumption; I=Investments; EX=Export; IMP=Imports; $RHPI$=Real disposable income of households; YE=Wages, salaries and self-employed income; BAL=Current deficit; $BALT$=Balance of trade; REX=Real exchange rate set equal to 1 in 2000; EER=The price of foreign currencies, index, set equal to 1 in 2000 (In year 2000 the official index of the price of foreign currencies published by the Central Bank of Iceland was 112.7); NFA=Net foreign assets; NFW=Net financial assets of households; PC=The price of private consumption.

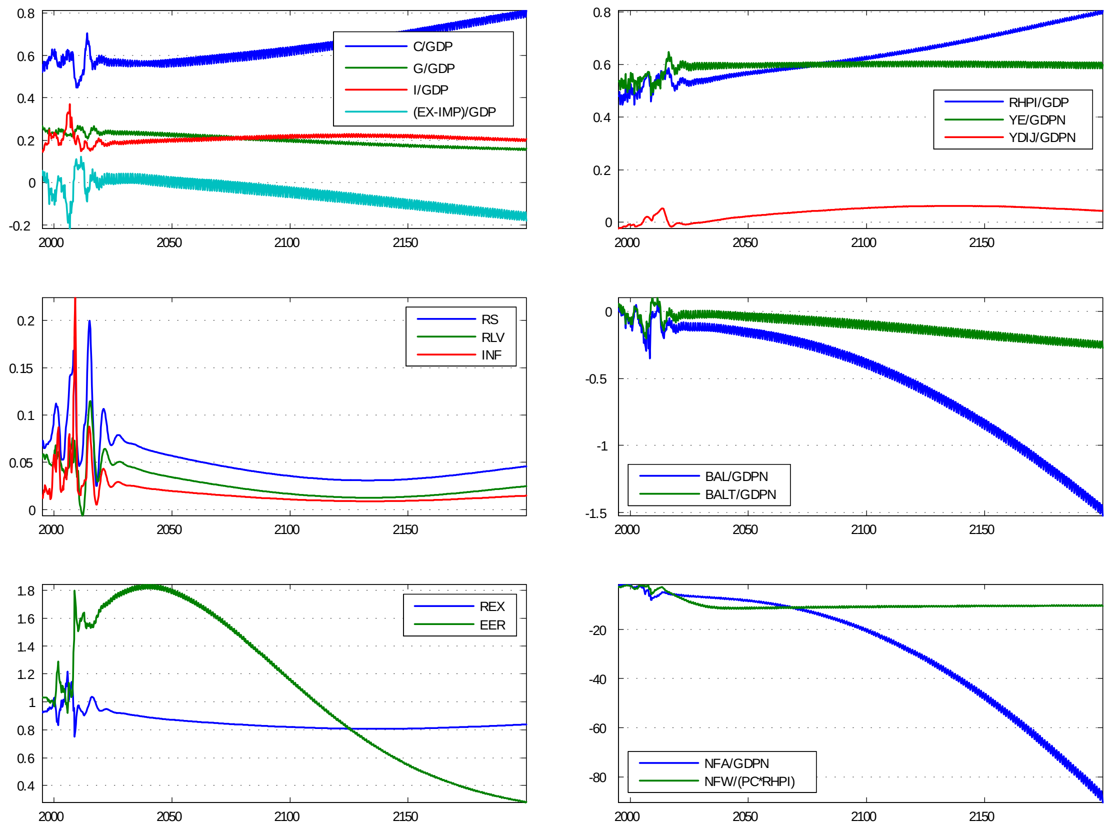

Figure 2.1: Results from long run simulations with QMM, starting in 2008Q3 and ending in 2200Q1

foreign debt to GDP ($NFA/GDPN$). This decrease, however, does not stop. If the increment in the risk premium for a given change in the level of foreign indebtedness is increased gradually, the downward slide in the ratio of foreign indebtedness to GDP decreases at first but quite soon the model starts to produce oscillations which become larger and larger until it eventually crashes indicating that something else must be done if the model is to able to simulate realistic lon-run paths.

3. Solving for the balanced growth path for QMM

3.1. General discussion of the problem

After suitable reformulations,[5] most endogenous variables in QMM can be classified into three groups: 1) real variables that grow at some constant rate on a balanced

[5]To simplify, it is used that nominal variables in QMM can be expressed as product of a real variable and its price. In some cases the relevant price is not explicit in QMM. In such cases the formulation below assumes an appropriate reformulation of the relevant equation in QMM.

growth path; 2) price variables that increase by the assumed equilibrium rate of inflation keeping all relative prices constant; and 3) variables that are constant on the equilibrium growth path. The last group contains all interest rates, nominal and real, and the real exchange rate and the rate of unemployment.

There are two endogenous variables in QMM that do not automatically belong to any of the three groups above. These are: 1) the nominal exchange rate which in equilibrium must change so as to maintain the constant real exchange rate which means that its change in each period must be equal to the difference between the domestic and foreign rates of inflation; and 2) the nominal wage rate which in equilibrium must increase by the sum of the rate of inflation and the rate of growth of labour productivity.[6] Assuming that the rate of inflation abroad is the same as the equilibrium rate of inflation in Iceland makes the equilibrium nominal exchange rate constant. It is also possible to formulate the model in terms of unit labour cost rather than the wage rate to avoid the second problem.

To avoid making the notation still more messy, the discussion in this section focuses on the backward looking version of QMM. In this case it is possible to write QMM as follows:[7]

$$X_t^n = Q_1\left(\left[X_t^n; X_{t-1}^n; X_t^x; P_t^n; P_{t-1}^n; P_t^x; R_t^n; R_{t-1}^n; R_t^x\right]\right) \tag{3.1}$$
$$P_t^n = Q_2\left(\left[X_t^n; X_{t-1}^n; X_t^x; P_t^n; P_{t-1}^n; P_t^x; R_t^n; R_{t-1}^n; R_t^x\right]\right) \tag{3.2}$$
$$R_t^n = Q_3\left(\left[X_t^n; X_{t-1}^n; X_t^x; P_t^n; P_{t-1}^n; P_t^x; R_t^n; R_{t-1}^n; R_t^x\right]\right) \tag{3.3}$$

where the column vector $X_t^n = \left(X_{t,1}^n, X_{t,2}^n, ..., X_{t,n1}^n\right)'$ is a vector of the $n1$ endogenous real variables in the model including all lags up to (but not including) the maximum lag, X_t^x is the vector of all exogenous real variables including all lags, P_t^n and P_t^x are similarily vectors of all endogenous and exogenous price variables in the model and R_t^n and R_t^x are vectors of endogenous and exogenous variables that are constant on an equilibrium growth path, e.g. all interest rates and the real exchange rate. For this rewriting of QMM as a set of first order difference equations, a variable and an equation must be added to those listed in Daníelsson et al. (2009) for each case where a variable appears in QMM with a lag greater than 1. This way of writing models is frequently used today and is refered to as "state space" representations.

If there exists a balanced growth path where all real variables grow at some given rate, γ_q, and all prices are increasing at some given rate of inflation π_q, it follows from equations (3.1) - (3.3) that the following must be true:

$$\overline{X}_t^n = Q_1\left(\left[\overline{X}_t^n; \frac{\overline{X}_t^n}{1+\gamma_q}; X_t^x; \overline{P}_t^n; \frac{\overline{P}_t^n}{1+\pi_q}; P_t^x; \overline{R}_t^n; \overline{R}_t^n; R_t^x\right]\right) \tag{3.4}$$

[6] See Section 6.2 below.

[7] In each equation in $Q_i, i = 1, 2, 3$, only a few variables appear on the right hand side and in no case does the left hand side variable in period t appear on the right hand side, but some other variables dated in period t do appear in most cases.

$$\overline{P}^n_t = Q'_2\left(\left[\overline{X}^n_t; \frac{\overline{X}^n_t}{1+\gamma_q}; X^x_t; \overline{P}^n_t; \frac{\overline{P}^n_t}{1+\pi_q}; P^x_t; \overline{R}^n_t; \overline{R}^n_t; R^x_t\right]\right) \tag{3.5}$$

$$\overline{R}^n_t = Q_3\left(\left[\overline{X}^n_t; \frac{\overline{X}^n_t}{1+\gamma_q}; X^x_t; \overline{P}^n_t; \frac{\overline{P}^n_t}{1+\pi_q}; P^x_t; \overline{R}^n_t; \overline{R}^n_t; R^x_t\right]\right) \tag{3.6}$$

where the bar over the endogenous variables indicates equilibrium values. The name of the set of price functions has been changed from Q_2 to Q'_2 to indicate that the equations for consumer prices and several other prices in QMM have been dropped because they do not provide any restriction on relative prices in an equilibrium where the rate of inflation is constant. As only relative prices matter for agents' decisions in QMM the level of equilibrium prices can be determined arbitrarily. In the calculations below it is assumed that the price of GDP in equilibrium is equal to the actual price level. Here it is sufficient to assume that some such equation is included in Q'_2.

The system of equations in (3.4) - (3.6) has the same number of unknowns (endogenous variables) as there are equations. If the system has a unique solution for the endogenous variables in period t, the equations in QMM are formulated so that there exists a solution for $\overline{X}^n_{t+1}$, $\overline{P}^n_{t+1}$ and $\overline{R}^n_{t+1}$ to the system of equations in (3.4) - (3.6) formulated for period $t+1$ such that $\overline{X}^n_{t+1} = \left(1+\gamma_q\right)\overline{X}^n_t$, $\overline{P}^n_{t+1} = \left(1+\pi_q\right)\overline{P}^n_t$ and $\overline{R}^n_{t+1} = \overline{R}^n_t$ and so on for all later periods.

It is consistent with the results in the preceding section that efforts to find a solution to the equations in (3.4) - (3.6) were unsuccessful. To obtain a solution, rescaling constants must be added to some equations as they appear in QMM. To obtain realistic values on the external debt ratio and the household debt ratio, equations that ensure that this is the case are added to the system. Writing the conditions for a balanced growth solution for this model, QMM-BG, gives a system of equation which has a unique solution to all endogenous variables, including the rescaling factors. The resulting model is compatible with a balanced growth path by construction.

3.2. Examples of balanced growth compatible equations

The equations in the system (3.4) - (3.6) are constructed by first making assumptions concerning the growth rate of the real economy and the nominal rate of change in the prices. γ_q is the fixed quarterly growth rate of real variables while π is the annual rate of inflation and $\pi_q = (1+\pi)^{0.25} - 1$ is the quarterly rate. Ignoring for simplicity the centered seasonal dummies, and all other dummies as well, the consumption function in QMM is given as:[8]

$$\Delta c_t = \kappa_c + \lambda_c \Delta c_{t-4} - \eta_c \Delta UR_t + \zeta_c \Delta(wel_t - pc_t) - \psi_c ecm(c)_{t-1} \tag{3.7}$$

[8] Ignoring the seasonal variations on the balanced growth path should give results that are close to those obtained from a model using seasonally adjusted data.

where lower case letters indicate log of variables with the same upper case name, C is private consumption, UR is the unemployment rate, WEL is household sector total wealth, PC is the private consumption deflator, Δ is the difference operator and $ecm(c)$ is the error term from the estimated long-run relationship given by:

$$ecm(c)_t = c_t - \beta_c rhpi_t - (1-\beta_c)(wel - pc)_t + \phi_c RLV_t \tag{3.8}$$

where $RHPI$ is the real disposable income and RLV is the long-term real rate of interest.

On the balanced growth path all real variables are growing at the constant rate of γ_q, i.e. $\Delta c_t = \Delta c_{t-4} = l\gamma_q = \log\left(1+\gamma_q\right)$ but the rate of unemployment is constant and therefore $\Delta UR_t = 0$. Substituting this into 3.8 and then into 3.7, simplifying, and using the superscript s to indicate that it is a value on the balanced growth path, gives the following equation for $c_t^s = \log\left(C_t^s\right)$:

$$\begin{aligned} c_t^s \quad = \quad & \left[\left(1-\lambda_c-\zeta_c\right)l\gamma_q - \kappa_c\right]/\left(-\psi_c\right) + \beta_c rhpi_t^s \\ & +(1-\beta_c)(wel-pc)_t^s - \phi_c RLV_t^s \end{aligned} \tag{3.9}$$

It is possible to manipulate other equations for endogenous real variables in QMM in a similar manner to obtain necessary conditions for the model to remain on a balanced growth path.

Some price equations in QMM give restrictions on the blanced growth path with constant rate of inflation, e.g. the equation for house prices which is:

$$\begin{aligned} \Delta(ph_t - cpi_t) \quad = \quad & \kappa_{ph} + \lambda_{ph}\Delta(ph_{t-2} - cpi_{t-2}) + \eta_{ph}\Delta ly_t \\ & +\zeta_{ph}\Delta RLV_t + \vartheta_{ph}\left[(ph_{t-2} - cpi_{t-2})\right. \\ & \left. +\beta_{ph}(kh_{t-2} - ly_{t-2}) + \phi_{ph} RLV_{t-2}\right] \end{aligned} \tag{3.10}$$

where, as before, lower case letters inticate logs, PH is the house price, CPI is the CPI-index, LY is real post-tax labour income and KH is the stock of housing capital. For this equation to be consistent with a constant growth path, where all real variables are growing at the constant rate of γ_q and all price variables change at the same constant rate the following must be true: $\Delta(ph_t - cpi_t) = \Delta(ph_t - cpi_t) = 0$, $\Delta RLV_t = 0$ and $\Delta ly_t = l\gamma_q$. Substituting this into 3.10 gives:

$$\begin{aligned} 0 \quad = \quad & \kappa_{ph} + \eta_{ph} l\gamma_q - \vartheta_{ph}\left[(ph_{t-2} - cpi_{t-2})\right. \\ & \left. +\beta_{ph}(kh_{t-2} - ly_{t-2}) + \phi_{ph} RLV_{t-2}\right] \end{aligned} \tag{3.11}$$

Moving this equation two periods forward and eliminating ph_t^s on the left hand side gives:

$$ph_t^s = cpi_t^s + \left(\eta_{ph}\gamma_q + \kappa_{ph}\right)/\vartheta_{ph} - \beta_{ph}(kh_t^s - ly_t^s) - \phi_{ph} RLV_t^s$$

In other cases, the price equations in QMM do not give any restriction for values on endogenous variables on the constant growth path with constant rate of inflation. This is e.g. the case with the equation for import prices (PM):

$$\begin{aligned}\Delta pm_t &= \alpha_{pm} pm_{t-1} + \lambda_{pm}\Delta(wpx_t + eer_t) + \theta_{pm}\Delta(wpx_{t-1} + eer_{t-1}) \\ &\quad +\eta_{pm}\Delta(pcom_t + eus_t) + \zeta_{pm}\Delta ulct_{t-2} \\ &\quad +(1-\alpha_{pm}-\lambda_{pm}-\theta_{pm}-\eta_{pm}-\zeta_{pm})\Delta(poil_{t-1} + eus_{t-1})\end{aligned} \tag{3.12}$$

where WPX is world export price in foreign currency, $PCOM$ is non-oil commodity prices in US dollars, $ULCT$ is the trend unit labour cost, $POIL$ is world oil price in US dollars, EER is the index of nominal effective exchange rate of foreign currencies and EUS is the index of US dollar exchange rate. This equation is homogenous in rates of change in exogenous and endogenous prices. This ensures that the equation allows that all these prices change at any constant rate. It is therefore possible to determine the equilibrium relative price of import exogenously. The relative prices for other price variables in QMM that are determined by homogenous price equations are obtained in the same way.

4. Some important relationships in QMM

It is instructive to consider some key equations in QMM-BG to get some intuition for some central features of the model. Consider first the CPI-equation.

4.1. The CPI-equation

The equation for consumer prices in QMM is:

$$\begin{aligned}\Delta_4 cpi_t &= \lambda_{cpi}\Delta_4 cpi_{t-1} + \eta_{cpi}\Delta_4 cpi_{t+8} \\ &\quad +(1-\lambda_{cpi}-\eta_{cpi})\log(1+IT_t) \\ &\quad +\zeta_{cpi}\Delta_4 rexm_{t-1} + \vartheta_{cpi}\Delta_4(ulct_{t-4} - cpi_{t-4}) + \psi_{cpi} GAPAV_{t-1}\end{aligned} \tag{4.1}$$

where IT is the Central bank's inflation target, $REXM$ is the real exchange rate, $ULCT$ is the trend unit labour cost and $GAPAV$ is the four quarter average output gap. On the balanced growth path where $\Delta_4 cpi_t = \Delta_4 cpi_{t-1} = \Delta_4 cpi_{t+8} = \log(1+IT_t) = \Delta_4 ulct_{t-4} = 4 \times l\pi_q$ and $\Delta_4 rexm_{t-1} = 0$, this equation gives no restriction on the level of CPI but requires obviously that $GAPAV_t = 0$. This means that the actual GDP must be equal to potential GDP on the balanced growth path, i.e. $GDP_t^s = GDPT_t^s$.

4.2. The production function and the marginal productivity conditions

As the rate of unemployment is equal to the equilibrium rate ($NAIRU$) on the balanced growth path and the participation rate (PA) is equal to its long-run equilibrium, which is constant, actual employment and trend employment are equal, $EMP_t^s = EMPT_t^s = PA_t^s \left(1 - NAIRU_t^s\right) POWA_t^s$. This means that in equilibrium growth of employment is determined by growth of population ($POWA$).

On the balanced growth path the production function in QMM, which gives trend productivity, is:

$$GDP_t^s = GDPT_t^s = \alpha_g \left(1 + \gamma_g\right)^{t \times \beta_g} \left(EMP_t^s\right)^{\beta_g} \left(K_t^s\right)^{1-\beta_g} \tag{4.2}$$

where K is the capital stock, γ_g is the constant rate of growth of labour productivity, β_g is the wage share and α_g is a constant that depends on the units chosen for volume of labour and labour productivity in period $t = 0$.

Economic theory assumes that the capital-output ratio is determined by the marginal productivity condition for capital:[9]

$$\begin{aligned} RCC_t^s \times PK_t^s \times K_t^s &= \left(1 - \beta_g\right) \times PGDP_t^s \times GDP_t^s \\ \Leftrightarrow \frac{K_t^s}{GDP_t^s} &= \frac{1 - \beta_g}{RCC_t^s} \times \frac{PGDP_t^s}{PK_t^s} \end{aligned} \tag{4.3}$$

where RCC is the real cost of capital. This equation is not part of QMM.

It will be argued below that it is difficult to reconcile the marginal productivity condition for capital with Icelandic data where the risk free real rate has been considerably above what is implied by the marginal productivity condition for capital. It is possible that this indicates that the data reflect some overvaluation of the capital stock, or that it is necessary to take the natural resource base of the Icelandic economy into account when evaluating the marginal productivity condition for capital. In the latter case it would though be necessary to add to the capital stock the considerable value of the natural resources, including the fishing rights that have commanded an high price in recent years.

It will be assumed below that the the equilibrium capital-output ratio, K_t^s/GDP_t^s, is given by historical data rather than by the marginal productivity condition for capital. Substituting this ratio, which is constant on a balanced growt path, into (4.2) gives:

[9] The variable PK_t^s, the price of total fixed capital is not explicit in QMM, but can be defined as follows in terms of the price of business investments, $PIBUS_t^s = IBUSN/IBUS$, which is not explicit in QMM either, and some variables explicit in the model: $PK_t^s K_t^s = PH_t^s KH_t^s + PIBUS_t^s KBUS_t^s + PIG_t^s \left(K_t^s - KH_t^s - KBUS_t^s\right)$.

Here K_t^s is the stock of capital at fixed price, PH_t^s is the price of housing, KH_t^s is the stock of housing capital, $KBUS_t^s$ is the stock of business capital, and PIG_t^s is the price of public investments.

$$
\begin{aligned}
GDP_t^s &= \alpha_g \left(1+\gamma_g\right)^{T\times\beta_g} \left(EMP_t^s\right)^{\beta_g} \left(\frac{K_t^s}{GDP_t^s}\right)^{1-\beta_g} \left(GDP_t^s\right)^{1-\beta_g} \\
&\Leftrightarrow \; GDP_t^s = \alpha_g^{1/\beta_g} \left(1+\gamma_g\right)^T \\
&\quad \times PA_t^s \left(1-NAIRU_t^s\right) POWA_t^s \left(\frac{K_t^s}{GDP_t^s}\right)^{\left(1-\beta_g\right)/\beta_g} \qquad (4.4)
\end{aligned}
$$

It follows from this that on the balanced growth path, GDP_t^s will grow at a rate which is the sum of γ_g and the growth rate of $POWA_t^s$, γ_n, i.e. the growth rate of the economy will be: $\gamma_q = \left(1+\gamma_g\right)\left(1+\gamma_n\right)-1 \;\; (\approx \gamma_g+\gamma_n)$.

The marginal productivity condition for labour is:

$$
W_t^s \times EMP_t^s = \beta_g PGDP_t^s GDP_t^s \qquad (4.5)
$$

Combining this with (4.2) above gives:

$$
\frac{W_t^s}{PGDP_t^s} = \beta_g \alpha_g^{1/\beta_g} \left(1+\gamma_g\right)^T \left(\frac{K_t^s}{GDP_t^s}\right)^{\left(1-\beta_g\right)/\beta_g} \qquad (4.6)
$$

This shows that on the balanced growth path where the capital-output ratio is constant, equilibrium real wage increases at the rate of technical progress $\left(\gamma_g\right)$ and the nominal wage rate grows at the rate of $\left(1+\gamma_g\right)\left(1+\pi_q\right)-1$ $(\approx \gamma_g+\pi_q)$. All other nominal prices grow at the constant rate of π_q on the balanced growth path.

4.3. Condition for the external balance

Finally, consider the equilibrium conditions for the external balance and net foreign debt. The ratio of net foreign assets (NFA) to nominal GDP is constant on the balanced growth. We will label this ratio h_{nfa}, i.e.:

$$
h_{nfa} = \frac{NFA_t^s}{GDPN_t^s} \qquad (4.7)
$$

The change in the net foreign assets of the country equals the current account balance (BAL) in each period, i.e.:

$$
NFA_t = NFA_{t-1} + BAL_t \qquad (4.8)
$$

On the balanced growth path NFA must grow at the same rate as all other nominal variables, i.e. at the rate of $\gamma_q = \left(1+\gamma_q^n\right)\left(1+\pi_q\right)-1$. This gives the following relationship between NFA_t^s and BAL_t^s:

$$
NFA_t^s = \left(\frac{1+\gamma_q^n}{\gamma_q^n}\right) BAL_t^s \qquad (4.9)
$$

and the following one between the balanced growth ratio of BAL to nominal GDP and h_{nfa}:

$$\frac{BAL_t^s}{GDPN_t^s} = \left(\frac{\gamma_q^n}{1+\gamma_q^n}\right) h_{nfa} \tag{4.10}$$

The current account balance is given by:

$$BAL_t = BALT_t + BIPD_t + BTRF_t \tag{4.11}$$

where $BALT$ is the trade balance, $BIPD$ is the balance of interest, salaries, dividends and profits, and $BTRF$ is the exogenous balance of transfers.

$BIPD$ is given by formula:

$$BIPD_t = \left(\frac{WRS_t + BIPDF_t}{4}\right) \frac{EER_t}{2} \left(\frac{NFA_t}{EER_t} + \frac{NFA_{t-1}}{EER_{t-1}}\right) \tag{4.12}$$

where WRS is short-term world rate of interest and $BIPDF$ is the risk premium. Hence, the steady state value is given as:

$$\frac{BIPD_t^s}{GDPN_t^s} = \left(\frac{WRS_t^s + BIPDF_t^s}{8}\right) \left(\frac{2+\gamma_q^n}{1+\gamma_q^n}\right) h_{nfa} \tag{4.13}$$

The equilibrium trade balance can therefore be obtained from (4.10) and (4.13) as:

$$\frac{BALT_t^s}{GDPN_t^s} = \left(\frac{1}{1+\gamma_q^n}\right) \left[\gamma_q^n - \left(\frac{WRS_t^s + BIPDF_t^s}{8}\right)(2+\gamma_q^n)\right] h_{nfa} - h_{btrf} \tag{4.14}$$

where h_{btrf} is the exogenously given ratio of balance of transfers ($BTRF_t$) to $GDPN_t^s$ on the balanced growth path.

The equation for the capital stock in QMM is:[10] $K_t = (1 - DELTA)\,K_{t-1} + I_t$ where I_t is gross investment and $DELTA$ is the rate of depreciations. As $K_t^s = \left(1+\gamma_q\right) K_{t-1}^s$ on the balanced growth path, the following must be true on this path:

$$I_t^s = \left[1 - \frac{1-DELTA}{1+\gamma_q}\right] K_t^s = \frac{\gamma_q + DELTA}{1+\gamma_q} K_t^s \tag{4.15}$$

i.e. if $\frac{K_t^s}{GDP_t^s}$ is assumed given then $\frac{I_t^s}{GDP_t^s}$ is also given. Public consumption is exogenous in QMM so it is natural to assume that $\frac{G_t^s}{GDP_t^s}$ is equal to an exogenously given constant on the balanced growth path. Stockbuilding is given in QMM as a function of the exogenous variables $EXMAR_t$ and $EXALU_t$ so that it is natural to assume that $\frac{II_t^s}{GDP_t^s}$ is also exogenously given constant. If the restriction on $\frac{NFA_t^s}{GDPN_t^s}$ gives the ratio

$$\frac{BALT_t^s}{GDPN_t^s} = \frac{PX_t^s \times EX_t^s}{GDPN_t^s} - \frac{PM_t^s \times IMP_t^s}{GDPN_t^s} \tag{4.16}$$

[10]Equation (5.18) in the handbook for QMM, Danielsson et al. (2009).

on the balanced growth path, the following equation, which is a national accounting identity,

$$\frac{C_t^s}{GDP_t^s} = 1 - \frac{G_t^s}{GDP_t^s} - \frac{I_t^s}{GDP_t^s} - \frac{II_t^s}{GDP_t^s} - \left[\frac{EX_t^s}{GDP_t^s} - \frac{IMP_t^s}{GDP_t^s}\right] \tag{4.17}$$

makes the set of possible solutions for the ratio $\frac{C_t^s}{GDP_t^s}$ fairly small and most of them are well below what has been the case in Iceland. This indicates that one of the reasons why it is not possible to obtain a balanced growth solution to QMM without rescaling some of its equations is that the estimated consumption equation in QMM implies a larger share for consumption in GDP than this equilibrium condition allows.

5. Routes to solve for variables in QMM-BG

The discussion above indicates that the equations in QMM-BG make it possible to solve for the equilibrium levels of the endogenous variables from a subset of equations in QMM-BG. Table 5.1 contains such a subset of equations which constitutes a determinate system when the capital-output ratio (K/GDP) and the equilibrium real rate (RRN) have been fixed. The latter variable is exogenous in QMM.

The equations in QMM are consistent with the assumption that agents make their decisions on the basis of relative prices. This makes the price level in a specific period t on the balanced growth path indeterminate. This degree of freedom is used to set the price level of GDP in period t on the balanced growth path, $PGDP_t^s$, equal to the value of this variable in the base period. If the exogenously given value of $h_{nfa} = \frac{NFA_t^s}{GDPN_t^s}$ is used, the value of $BALT_t^s$ is derived from (4.14) above and values of K_t^s and RRN_t^s are determined as described above, then the system of 37 equations in Table 5.1 can be used to solve for the 37 endogenous (left hand side) variables.

Table 5.1 contains equations derived from QMM, except the equation for constant capital-output ratio (4^{th} equation). The equation for the equilibrium real rate of interest (RRN) is not formally in QMM as it is also exogenously determined in that model. The price level of GDP ($PGDP$) can be determined arbitrarily on the balanced growth path as explained above.

To avoid explosive solutions it is reasonable to fix the ratio $h_{nfw} = \left(\frac{NFW}{GDPN}\right)^s$ on the balanced growth path. It also seems reasonable to adjust the tax parameter $ALLOW_t$ which is exogenous in QMM so that the share of taxes in labour income is equal to some exogenously given value. This is done by adding the equation $ALLOW_t^s = \frac{W_t}{W_0} ALLOW_0$ to the equations determining the balanced growth path for QMM-BG so the share of taxes on the balanced growth path is equal to the share of taxes in the base period, t=0.

The system of equations in Table 5.1 makes it possible to obtain a solution for private consumption (C_t^s) without using the balanced growth version of the equation

Table 5.1: Solvable subset of equations

Endog.(lhs) variable	Endog.(rhs) variable(s)	no. of eq. in QMM
PA^s_t		(6.7)
EMP^s_t	PA^s_t	(6.9)
GDP^s_t	EMP^s_t	(5.42), the prod. function
K^s_t	GDP^s_t	K^s_t/GDP^s_t=const., new eq.
RRN^s_t		Exogenous in QMM
RS^s_t	RRN^s_t	(4.1)/(4.2)
RL^s_t	RS^s_t	(4.3)
RLV^s_t	RL^s_t	(4.4)
RCC^s_t	RLV^s_t	(4.5)
I^s_t	K^s_t	(5.18)
II^s_t		(5.21)
DD^s_t	GDP^s_t, EX^s_t, IMP^s_t	(5.39)
C^s_t	DD^s_t, I^s_t, II^s_t	(5.23)
$PGDP^s_t$		Fixed
$GDPN^s_t$	$PGDP^s_t$, GDP^s_t	(7.10)
$BALT^s_t$	$GDPN^s_t$	Eq. from (5.34)-(5.38)
IMP^s_t	$REXM^s_t$, DD^s_t	(5.30)
PM^s_t	$ULCT^s_t$, EER^s_t, EUS^s_t	(7.2)
$PXREG^s_t$	CPI^s_t, EER^s_t	(7.3)
$IMPN^s_t$	PM^s_t, IMP^s_t	(5.31)
EXN^s_t	$BALT^s_t$, $IMPN^s_t$	(5.33)
$EXREG^s_t$	EUS^s_t, EER^s_t, $PXREG^s_t$	(5.28)
REX^s_t	$EXREG^s_t$	(5.26)
$REXM^s_t$	PM^s_t, $PGDP^s_t$	(4.11)
EX^s_t	$EXREG^s_t$	(5.27)
EER^s_t	CPI^s_t, REX^s_t	(4.6)
EUS^s_t	EER^s_t	(4.7)
PI^s_t	BC^s_t, PM^s_t	(7.7)
PG^s_t	$ULCT^s_t$, CPI^s_t	(7.6)
IN^s_t	PI^s_t, I^s_t	(5.6)
GN^s_t	G^s_t	(5.4)
IIN^s_t	EER^s_t, EUS^s_t	(5.22)
CN^s_t	$GDPN^s_t$, IN^s_t, GN^s_t, IIN^s_t, $BALT^s_t$	(5.24) and (5.40)
PC^s_t	CN^s_t, C^s_t	(5.3)
CPI^s_t	PC^s_t	(7.5)
$ULCT^s_t$	REX^s_t, $PGDP^s_t$	(6.2)
BC^s_t	CPI^s_t, $ULCT^s_t$	(7.13)

for private consumption in QMM.[11] It follows that it is in most cases necessary to interfer with the restriction that follows from the balanced growth version of this equation. This is done by introducing the variable $hpC = C^s_0/C^*_0$ where C^s_0 is the balanced growth solution to the system of equations in Table 5.1 and C^*_0 is the solution to the balanced growth version of the equation for private consumption in QMM using the solutions from the system of equations in Table 5.1 for all other endogenous variables. The subscript 0 refers to the base period.

By adding the 25 equations in Table 5.2 below to those in Table 5.1 gives a solvable subsystem. Below we will discuss this system of 62 equations.

The consumption of housing services is included in private consumption. But in QMM there is an independent equation for investment in housing (IH_t) and another for the level of housing capital (KH^s_t),[12] the determinant of the volume of housing services supplied (and consumed). It therefore seems reasonable that if the level of

[11] Equation (5.2) in Danielsson et al. (2009).

[12] Investment in housing is given by equation (5.14), and the level of housing capital by equation (5.20) in Danielsson et al. (2009).

Table 5.2: Additional equations that can be solved with those in Table 5.1

Endog.(lhs) variable	Endog.(rhs) variable(s)	no. of eq. in QMM
$IBAIR^s_t + IBALU^s_t + IG^s_t$	I^s_t, IH^s_t, $IBREG^s_t$	(5.5) and (5.11)
IH^s_t	KH^s_t	(5.20)
$IBREG^s_t$	GDP^s_t, RCC^s_t	(5.10)
PH^s_t	CPI^s_t, KH^s_t, LY^s_t, RLV^s_t	(7.12)
PIH^s_t	BC^s_t	(7.8)
KH^s_t	CN^s_t, PH^s_t	New eq.*
W^s_t	$ULCT^s_t$, $PROD^s_t$	(6.4)
$PROD^s_t$	GDP^s_t, EMP^s_t	(6.11)
YE^s_t	W^s_t, EMP^s_t	(9.2)
YIC^s_t	YE^s_t	(9.3)
CJ^s_t	$GDPN^s_t$, $GAPAV^s_t$	(8.16)
NFW^s_t	$RHPI^s_t$, PC^s_t	New eq.†
$YDIJ^s_t$	$GDPN^s_t$, RLV^s_t, RS^s_t, CPI^s_t	(9.4)
$ALLOW^s_t$	W^s_t	New eq.‡
TJY^s_t	YE^s_t, CJ^s_t, $ALLOW^s_t$	(8.3)
TJO^s_t	$GDPN^s_t$	(8.5)
TI^s_t	$GDPN^s_t$, EQP^s_t, RL^s_t	(8.4)
EQP^s_t	$PGDP^s_t$	(4.19)
LY^s_t	YJ^s_t, TJ^s_t, PC^s_t	(9.6)
YJ^s_t	YE^s_t, CJ^s_t, YIC^s_t, $YIDJ^s_t$	(9.1)
TJ^s_t	TJY^s_t, TJO^s_t, TI^s_t	(8.2)
$RHPI^s_t$	PC^s_t, CN^s_t, PH^s_t, KH^s_t, IH^s_t, NFW^s_t	(4.22)-(4.24)
WEL^s_t	PH^s_t, KH^s_t, NFW^s_t	(4.20) and (4.21)
hpC	WEL^s_t, PC^s_t, RLV^s_t, $RHPI^s_t$, C^s_t	(5.2)
$hpIH$	IH^s_t, GDP^s_t, PH^s_t, PIH^s_t	(5.14)

Notes: *New eq.: $KH^s_t = hpKH0 \cdot CN^s_t/PH^s_t$. $hpKH0$ *is exog. fixed based on data*

†$hpNFW$ is exogenously fixed on the basis of data on $NFW_t/(PC_t \cdot RHPI_t)$

‡New eq.: $ALLOW^s_t = ALLOW0 \cdot W^s_t/W0$. ALLOW is exogenous in QMM

overall consumption on the balanced growth path is adjusted in some manner, the level of housing capital should be adjusted similarily. Consumption in QMM is not derived from maximisation of an utility function which would have made it possible to obtain a solution for the optimal mix of consumption of housing services and other consumption on the balanced growth path with a given set of prices. The solution here is to add an equation for the housing stock on the balanced growth path. This equation is based on the familiar result from consumer theory where the the consumer maximizes a Cobb-Douglas utility function. In this case the expenduture shares are constant so that:

$$KH^s_t = hpKH0 \times CN^s_t/PH^s_t \tag{5.1}$$

where $hpKH0 = PH_0 \times KH_0/CN_0$, the subscript t indicates the value of an variable in period t where the economy is on the balanced growth path while the subscript 0 indicates its value the base period. As IH^s_t is given by a balanced growth version of the equation for KH_t in QMM it is necessary to introduce a scaling constant in the equation for IH_t in QMM-BG in the same manner as was done for the consumption function above. The new variable is $hpIH = IH^s_0/IH^*_0$ where IH^*_0 is the solution to the balanced growth version of the equation in QMM using the solutions from the system of equations in Tables 5.1 and 5.2 for all other endogenous variables.

The value of real disposable income that is compatible with balanced growth ($RHPI^s_t$) is derived from the equilibrium condition for the asset account of the household sector in QMM.[13] This is necesary in most cases and can be justified on the grounds that there is great uncertainty concerning the relationship between the income account and the asset account for the households in Iceland. This way of deriving the equilibrium value for $RHPI_t^s$ means that it generally differs from what the equation for $RHPI_t$ in in QMM give,[14] i.e.:

$$RHPI_t = \left(\frac{YJ_t - TJ_t}{PC_t}\right) \tag{5.2}$$

where YJ_t is total income of households, TJ_t is the sum of all taxes to be paid and PC_t is the price level of private consumption.

In the simulations with QMM-BG below, the equation

$$RHPI_t = RHPI_{t-1}\left(\frac{YJ_t - TJ_t}{PC_t}\right) / \left(\frac{YJ_{t-1} - TJ_{t-1}}{PC_{t-1}}\right)$$

will be used for the real disposable income. This is, of course, equivalent to multiplying the right hand side of equation (5.2) with a constant. This scaling of the real disposable income affects the scaling that is necessary in the consumption function, i.e. it affects the value of hpC which was defined above.

The balanced growth compatible versions of the functions for private consumption (C_t), total household wealth (WEL_t) and housing wealth (HW_t) in QMM give that:[15]

$$\frac{C_t^s}{RHPI_t^s} = \phi\left[\frac{NFW_t^s + PH_t^s \times KH_t^s}{PC_t^s \times RHPI_t^s}\right]^{\lambda} \times \left(RLV_t^s\right)^{-\eta} \tag{5.3}$$

where $\phi = \exp\left(\left[(1 - 0.384071 - 0.245512) \times l\gamma_q - 0.089276\right] / (-0.344975)\right)$, $\lambda =$ 0.0313 and $\eta = 2.431591$.

Using the balanced growth relationship between housing investment and housing capital[16] makes it possible to rewrite the balanced growth version of the equation for net financial wealth of households can be written as follows:[17]

$$\frac{NFW_t^s}{PC_t^s \times RHPI_t^s} = \frac{1+\gamma_q}{\gamma_q}\left[1 - \frac{PC_t^s \times C_t^s + \frac{\gamma_q}{1+\gamma_q} PH_t^s \times KH_t^s}{PC_t^s \times RHPI_t^s}\right] \tag{5.4}$$

If the left hand side of equation (5.4) is a constant and therefore:

[13] I.e. equations (4.22)-(4.24) in Danielsson et al. (2009).

[14] Equations (9.5) in Danielsson et al. (2009).

[15] The consumption function is given by equation (5.2) in Danielsson et al. (2009), WEL_t is given by equation (4.20) and HW_t by equation (4.21).

[16] Equation (5.20) in Danielsson et al. (2009), which on the balance growth path becomes: $IH_t = \frac{\gamma_q + DELTAH}{1+\gamma_q} KH_t$.

[17] Equation (4.20)-(4.22) in Danielsson et al. (2009) give that on the balance growth path: $NFW_t = \frac{1+\gamma_q}{\gamma_q}\left[PC_t \times RHPI_t - (CN_t + PH_t \times (IH_t - DELTAH_t \times KH_{t-1}))\right]$. Substituting for IH_t and simplifying gives: $NFW_t = \frac{1+\gamma_q}{\gamma_q}\left[PC_t \times RHPI_t - \left(CN_t + PH_t \frac{\gamma_q}{1+\gamma_q} KH_t\right)\right] = \frac{1+\gamma_q}{\gamma_q}\left[PC_t \times RHPI_t - CN_t\right] - PH_t \times KH_t$.

$$\frac{PC_t^s \times C_t^s + \frac{\gamma_q}{1+\gamma_q} PH_t^s \times KH_t^s}{PC_t^s \times RHPI_t^s}$$

is a constant, equation (5.3) above gives that if C_t^s changes for some reason, then $RHPI_t^s$ must change proportionally.

It is of course possible to allow that the equation above determines the equilibrium level of net financial wealth of households, $\frac{NFW_t^s}{PC_t^s \times RHPI_t^s}$. The problem with this option is that the level of $RHPI_t^s$ is not well determined which means that equation (5.4) frequently gives implausible equilibrium values for $\frac{NFW_t^s}{PC_t^s \times RHPI_t^s}$. For these reasons it was decided to fix $\frac{NFW_t^s}{PC_t^s \times RHPI_t^s}$ at some realistic value and then adjust the level of $RHPI_t^s$ so that equation (5.4) holds.

$IBAIR_t + IBALU_t + IG_t$, where $IBAIR_t$ is investment in airoplanes, $IBALU_t$ is aluminium sector investment, including investment in energy production and IG_t is public investment, is exogenous in QMM. Total investment, I_t, is given by:

$$I_t = IBREG_t + IH_t + (IBAIR_t + IBALU_t + IG_t) \tag{5.5}$$

The capital-output ratio was used to determine the balanced growth value of investment, I_t^s, and equations in Table 5.2 above provide values on $IBREG_t^s$ and IH_t^s.

It follows directly from Equation (5.5) that:

$$IBAIR_t^s + IBALU_t^s + IG_t^s = I_t^s - IBREG_t^s - IH_t^s \tag{5.6}$$

i.e. the exogenous variable, $IBAIR_t^s + IBALU_t^s + IG_t^s$, must be equal to the value of endogenous variables that have already been solved for. The system is therefore overdeterminate and it is necessary to relax some restriction. Here it is done by making the sum of the exogenous variables, $IBAIR_t^s + IBALU_t^s + IG_t^s$, endogenous as shown in Table 5.2. Another way would be to relax assumptions concerning the equilibrium real rate, the capital-output ratio and/or the marginal productivity condition for capital. Introducing an exogenous value for $\left(IBAIR_t^s + IBALU_t^s + IG_t^s\right)/GDP_t^s$ and using the marginal productivity condition for capital, the system of equations in Tables 5.1-5.2 gives a solution to the equilibrium real rate, RRN_t^s, and the capital-output ratio on the balanced growth path. The realism of the endogenously determined values on $(IBAIR_t^s + IBALU_t^s + IG_t^s)$ for different values on RRN_t^s will be used below in the discussion of the proper value for RRN_t^s.

The solution to the balanced growth path for QMM-BG, described above, gives values for the endogenous variables of the model on an imaginary balanced growth path. The exogenously given value of $POWA_t^s$ determines the level of employment together with the real rate of interest and the capital-output ratio determines the level of production while the exogenously (and arbitrarily) given value on $PGDP_t^s$ pins down the price system. Various exogenous variables, like export of marine products and aluminium and the prices of these goods affect the solutions for the exchange rate and other endogenous variables. The next section discusses assumptions concerning the exogenous variables of the model, i.e. the calibration of the long-run values for the relevant exogenous variables for QMM-BG.

6. Calibration of QMM-BG

6.1. Appropriate (long-run) values of the exogenous variables

On the balanced growth path, both marine export ($EXMAR_t^s$) and aluminium export ($EXALU_t^s$) must grow at the constant rate of γ_q[18] and the real exchange rate must be constant. If domestic price variables are assumed to grow at the rate of π_q, foreign prices must grow at this same rate in terms of domestic currency. This means that if exogenous foreign prices are assumed to grow at the rate of π_q^f then $\pi_q = \left(1+\pi_q^f\right) EER_t^s/EER_{t-1}^s - 1$ on the balanced growth path and nominal exchange rate (EER_t^s) is given by $EER_t^s = EER_{t-1}^s \left(1+\pi_q\right) / \left(1+\pi_q^f\right)$.

The values of the endogenous variables on the balanced growth path depend on the values of the exogenous variables. If it is assumed that the price of aluminium ($PXALU_t^s$) is higher or the volume of marine export ($EXMAR_t^s$) is higher then the equilibrium real exchange rate (REX_t^s) is higher and so is the equilibrium nominal exchange rate (i.e. EER_t^s, the price of foreign currency is lower) etc.

To obtain values for the exogenous variables that can be considered as long-run values the average values over some period of time are chosen. The exogenous domestic prices are based on the average of the relative price against the price of GDP ($PGDP_t^s$), i.e. $P_t^s = PGDP_t \times \frac{1}{n}\sum_{i=1}^{n}\frac{P_i}{PGDP_i}$ where t is some specific period (here a year to avoid effects of seasonal factors). For foreign prices (PF_t^s) it is similarly assumed that $PF_t^s = WCPI_t \times \frac{1}{n}\sum_{i=1}^{n}\frac{PF_i}{WCPI_i}$ if the foreign price can be converted to ISK using EER_t^s, but $PF_t^s = \frac{EER_t WCPI_t}{EUS_t} \times \frac{1}{n}\sum_{i=1}^{n}\frac{PF_i EUS_i}{EER_i WCPI_i}$ if the foreign price is in USD. $WCPI_t$ is the index for world CPI and EUS_t is the index of the ISK/USD rate.

The nominal value of some exogenous variable XN_t^s on the balanced growth path is determined as $XN_t^s = GDPN_t \times \frac{1}{n}\sum_{i=1}^{n}\frac{XN_i}{GDPN_i}$ where $GDPN_i$ is the nominal GDP in period i. To obtain the values for the exogenous real variables on the balanced growth path the formula $X_t^s = GDP_t \times \frac{1}{n}\sum_{i=1}^{n}\frac{X_i}{GDP_i}$ is used. Table 6.1 describes the assumptions concerning the exogenous variables in greater detail.

Table 6.2 shows the assumptions concerning the values of some (relative) prices that have to be determined exogenously. The value of the ratio of net financial wealth of household to disposable income is chosen on the basis of historical values for the

[18] This assumption ignores that the marine export is based on the exploitation of finite renewable resources (the implications of this restriction is discussed in Lúdvík Elíasson, 2004). It also ignores the fact that investments in aluminium come as large chunks that cause large jumps in the export of aluminum at irregular intervals.

Table 6.1: Estimations of values for exogenous variables in QMM_BG

Exog.variable	Formula used	Period of time	Value
$ALLOW$	$W \cdot \overline{[ALLOW/W]}$	2001Q1-2006Q4	0.12468
$BIPDF$	$\overline{[BIPDF]}$	2001Q1-2006Q4	0.01278
$BTRF$	$GDPN \cdot \overline{[BTRF/GDPN]}$	2001Q1-2006Q4	-303.4
$DELTA$	Simple average	2001Q1-2006Q4	0.01001
$DELTAB$	Simple average	2001Q1-2006Q4	0.01492
$DELTAG$	Simple average	2001Q1-2006Q4	0.00844
$DELTAH$	Simple average	2001Q1-2006Q4	0.00612
$EXALU$	$GDP \cdot \overline{[EXALU/GDP]}$	2001Q1-2006Q4	11,861.5
$EXMAR$	$GDP \cdot \overline{[EXMAR/GDP]}$	2001Q1-2006Q4	29,298.4
G	$GDP \cdot \overline{[G/GDP]}$	2001Q1-2006Q4	51,957.4
$PCOM$	$((WCPI)/(EUS/EER)) \cdot \overline{[PCOM \cdot \frac{(EUS/EER)}{WCPI}]}$	2001Q1-2006Q4	1.25625
$POIL$	$((WCPI)/(EUS/EER)) \cdot \overline{[POIL \cdot \frac{(EUS/EER)}{WCPI}]}$	2001Q1-2006Q4	1.40475
$POWA$	Simple average	2006Q1-2006Q4	199,312
$PXALU$	$\frac{(WCPI)}{EUS/EER} \cdot \overline{[PXALU \cdot \frac{EUS/EER}{WCPI}]}$	2001Q1-2006Q4	1.17390
$PXMAR$	$WCPI \cdot \overline{[PXMAR/WCPI]}$	2001Q1-2006Q4	1.13503
RCI	Simple average	2006Q1-2006Q4	0.02737
RCP	Simple average	2006Q1-2006Q4	0.01548
REM	Simple average	2006Q1-2006Q4	1.21356
$RFIC$	Simple average	2006Q1-2006Q4	0.00520
RI	Simple average	2006Q1-2006Q4	0.10000
RIC	Simple average	2006Q1-2006Q4	0.11459
$RIMP$	Simple average	2006Q1-2006Q4	0.08282
RJO	Simple average	2006Q1-2006Q4	0.00061
RJY	Simple average	2006Q1-2006Q4	0.36720
RSD	Simple average	2004Q1-2006Q4	0.04631
RTS	Simple average	2004Q1-2006Q4	0.01347
$RVAT$	Simple average	2004Q1-2006Q4	0.18712
RWC	Simple average	2004Q1-2006Q4	0.05808
$TRADE$	$WGDP \cdot SPEC$		1.34709
$WCPI$	Simple average	2006Q1-2006Q4	1.13680
$WEQP$	$WCPI \cdot \overline{[WEQP/WCPI]}$	2001Q1-2006Q4	0.85319
$WGDP$	$GDP \cdot \overline{[WGDP/GDP]}$	2001Q1-2006Q4	1.21859
WPX	$WCPI \cdot \overline{[WPX/WCPI]}$	2001Q1-2006Q4	1.08155
WRS	Simple average	1994Q1-2006Q4	0.03864

Notes: $\overline{[X/Y]}$ denotes the average of X/Y.

period 2001Q1-2006Q4. The present turmoil in the financial markets in Iceland and internationally, makes the future long-run value for the rate of interest on foreign debt rather uncertain. This rate of interest $(WRS + BIPDF)$ is set equal to 7%, which is equal to the estimated risk free equilibrium nominal policy rate in Iceland. The present turmoil also creates uncertainty concerning the ratio of net foreign assets to GDP. Below this ratio is set equal to -4 (i.e. net foreign debt, NFA, is equal to annual nominal GDP).

6.2. Estimating the wage share

If the production function is homogenous of degree one in labour and capital and there is perfect competition so that there are no pure profits in equilibrium, then the value of the production must be equal to the incomes of the factors, i.e. $P_t Y_t =$

Table 6.2: Estimations of values for exogenous price variables in QMM_BG

Price index	Formula used	Period of time	Value
BC	$PGDP \cdot \overline{[BC/PGDP]}$	2001Q1-2006Q4	1.355
PG	$PGDP \cdot \overline{[PG/PGDP]}$	2001Q1-2006Q4	1.399
PI	$PGDP \cdot \overline{[PI/PGDP]}$	2001Q1-2006Q4	1.328
PIG	$PGDP \cdot \overline{[PIG/PGDP]}$	2001Q1-2006Q4	1.331
PIH	$PGDP \cdot \overline{[PIH/PGDP]}$	2001Q1-2006Q4	1.428
PM	$PGDP \cdot \overline{[PM/PGDP]}$	1994Q1-2006Q4	1.333
$PXREG$	$PGDP \cdot \overline{[PXREG/PGDP]}$	1994Q1-2006Q4	1.361

Notes: $\overline{[X/Y]}$ denotes the average of X/Y

$P_t F(K_{t-1}, L_t) = W_t L_t + (r_t + \delta_t) P_t^K K_{t-1}$.[19] This means that $P_t Y_t$ must be equal to the nominal value of the Gross Factor Income (GFI_t) which is considerably smaller than the nominal (market) value of GDP. The relationship between GFI_t and $GDPN_t$ is given by $GDPN_t = GFI_t + Ti_t - S_t$, where Ti_t is indirect taxes and S_t subsidies. The adding up property gives that $GDPN_t = W_t L_t + (r_t + \delta_t) P_t^K K_{t-1} + Ti_t - S_t$.

The correct way to estimate the parameter β in the Cobb-Douglas production function, $F(K_{t-1}, L_t) = A_t K_{t-1}^{1-\beta} L_t^{\beta}$, is to use $\widehat{\beta} = W_t L_t / GFI_t$ (**not** $\widehat{\beta} = W_t L_t / GDPN_t$). In terms of $GDPN_t$ the marginal productivity conditions are therefore: $\widehat{\beta} \times Sh_t^{GFI} GDPN_t / L_t = W_t$ and $\left(1 - \widehat{\beta}\right) Sh_t^{GFI} GDPN_t / \left(P_t^K K_{t-1}\right) = r_t + \delta_t$ where $Sh_t^{GFI} = GFI_t / GDPN_t$. It follows that $\frac{P_t^K \times K_{t-1}}{GDPN_t} = \frac{\left(1-\widehat{\beta}\right) Sh_t^{GFI}}{r_t + \delta_t}$ and that $r_t = \frac{\left(1-\widehat{\beta}\right) Sh_t^{GFI}}{P_t^K \times K_{t-1} / GDPN_t} - \delta_t$.

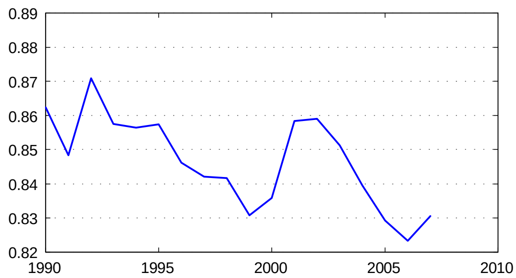

Figure 6.1: Share of Gross Factor Income in nominal GDP, GFI/GDPN, 1990-2007

Figure 6.1 shows actual values of $Sh_t^{GFI} = GFI_t / GDPN_t$ for the period 1990-2007. The figure indicates that there might be a trend in Sh_t^{GFI} but probaby most of what looks like a trend can be explained by the fact that indirect taxes vary procyclically. During the recession in early 1990s the share of indirect taxes was relatively small (Sh_t^{GFI} high) but their share increased as the rate of growth increased towards the end of the decade. A contraction in 2001 and 2002 caused another dip in

[19] Here the notation used in theoretical discussion of production functions is used instead of the notation in QMM. P is the price level, Y is output, K is capital, L is labour, W the wage rate, r the real rate of interest, δ is the rate of depreciations and P^K is the price of capital.

the weight of indirect taxed but they increased again during the expansion in recent years.

In QMM the wage rate (W) is an index. It is therefore not possible to set it equal to the value of the marginal productivity of labour ($\widehat{\beta} \times Sh_t^{GFI} GDPN_t/L_t$). Note also that the value of the marginal product should be equal to the cost of labour for the firms, i.e. employers' wage-related cost (REM) should be included.

Figure 6.2 shows the ratio $(W \times EMP)/YE$.[20] The data are from the income accounts produced by Statistics Iceland. $W \times EMP$ includes estimated (for tax purposes) wages of self-employed. The figure also shows the variable hlf_lye defined as:

$$hlf\_lye = (W \times EMP \times REM)/YE$$

Using this equation and data for 2000-2007 gives hlf_lye=1.115.

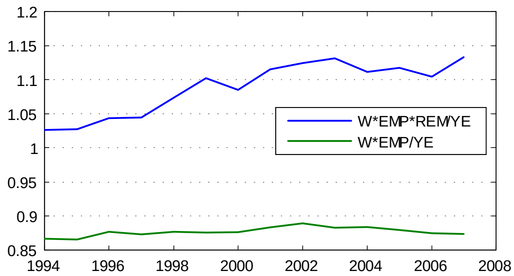

Figure 6.2: Indices of shares of wage cost and wage income 1994-2007

The marginal productivity condition for labour gives that

$$\begin{aligned} W_t \times REM_t &= \widehat{\beta} \times Sh_t^{GFI} GDPN_t/EMP_t \\ &\Leftrightarrow \; W_t = \widehat{\beta} \times Sh_t^{GFI} \frac{GDPN_t}{EMP_t \times REM_t} \end{aligned} \tag{6.1}$$

The difference between the actual value of the wage rate in a given time period, $W_0 = \frac{hlf\_lye \times YE_0}{EMP_0 \times REM_0}$, and the equilibrium value, $W_t = \widehat{\beta} \times Sh_t^{GFI} \frac{GDPN_t}{EMP_t \times REM_t}$, is then

$$\begin{aligned} \frac{W_t}{W_0} &= \frac{\widehat{\beta} \times Sh_t^{GFI} \times GDPN_t \times EMP_0 \times REM_0}{EMP_t \times REM_t \times hlf\_lye \times YE_0} \\ &= \frac{\widehat{\beta} \times Sh_t^{GFI} \times GDPN_t \times EMP_0}{EMP_t \times hlf\_lye \times YE_0} \end{aligned} \tag{6.2}$$

This equation gives the equilibrium condition for the wage rate when the marginal productivity condition for labour is used.

[20] EMP symbolises the volume of employment in QMM. YE is wages, salaries and self-employed income.

As $\Delta ye_t = \Delta w_t + \Delta emp_t$ in QMM:

$$\frac{YE_t}{YE_0} = \frac{W_t}{W_0} \frac{EMP_t}{EMP_0} \tag{6.3}$$

and therefore also:

$$\begin{aligned} \frac{YE_t}{YE_0} &= \frac{\widehat{\beta} \times Sh_t^{GFI} \times GDPN_t}{hlf\_lye \times YE_0} \\ \Leftrightarrow \; YE_t &= \frac{\widehat{\beta} \times Sh_t^{GFI} \times GDPN_t}{hlf\_lye} \end{aligned} \tag{6.4}$$

The conclusion from the arguments above is that $\widehat{\beta}$ should be estimated from the share of wage cost, estimated wages and wage related costs in Gross Factor Income. The data on this variable is plotted in Figure 6.3 below together with some other related variables.

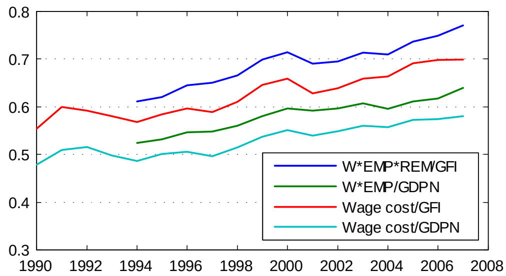

Figure 6.3: Share of wage cost in nominal GDP (GDPN) and Gross Factor Income (GFI)

Figure 6.3 shows wage cost, as it is estimated in Statistics Iceland's production accounts, as a share of GDP at market prices ($GDPN$) and as a share of GFI. The figure also shows the wage cost, including estimated salaries of self-employed persons, as estimated in Statistics Iceland's income accounts. It seems probable that the estimated salaries of self-employed persons is biased downwards. There exist some evidence indicating that the underestimation of the total wage cost may be increasing as more tax payers are exploiting the fact that presently profits are taxed at a lower rate than wages in Iceland. It is a bit problematic that the data in Figure 6.3 show a clear upward trend since the 1980s.[21]

Below, it will be assumed that the marginal productivity condition for labour determines the wage rate. It is also assumed that the equilibrium long-run level for

[21] It is worth noting that at the same time the share of wage cost in Western Europe shows a downward trend. Lawless and Whelan (2007), Table 5, p. 33 show that between 1979 and 2001 the labour share has declined in all European countries included except two, Luxembourg and Portugal. In Portulgal the share increases by 2.8 percentage points and in Luxembourg it increases by 1.7 percentage points. In Ireland the labour share decreases from 56.2% in 1979 to 45.9% in 2001. See also Stockhammer and Ederer (2008).

the wage share is below the most recent values but also above 0.64, the level used in QMM before version 2.0. It seems reasonable to choose the value of $\widehat{\beta} = 0.70$ and then calibrate

$$YE = \widehat{\beta} \times GDPN/\left(1 + hlf\_obsk\right)/\left(1 + hlf\_lye\right)$$

where $hlf\_obsk = 0.1859$ is the ratio of indirect taxes minus subsidies to Gross Factor Income (0.1859 is the average value during 2001Q1-2006Q4) and hlf_lye =1.115 as explained above.

6.3. Solving for the equilibrium real rate of interest

Even if the system of equations in (3.4) - (3.6) above has as many unknowns as there are equations it is quite possible that there is no solution, e.g. because the logarithmic function requires that its argument is positive. It is also possible that there exists a solution but it contains quite unreasonable values for some endogenous variables.

When constructing the system of equations for the QMM-BG, the capital-output ratio and also the equilibrium risk free interest rate were fixed exogenously. This means that the marginal condition for capital, which gives a determinate relationship between the capital-output ratio and the equilibrium real rate of interest, does not hold. As will be explained further below, the reason is that it doesn't seem to fit the data.

The relationship between the capital-output ratio and the equilibrium real rate (see equation (4.3)) involves the share of wage cost, $\widehat{\beta}$. In the previous section it was shown that the estimate of $\widehat{\beta}$ has been increasing and therefore the share of capital cost, $1 - \widehat{\beta}$, has been decreasing. Figure 6.3 shows that wage cost as a share of GFI has exceeded 70% in recent years. Here it will be assumed here that the reasonable long-run estimate of the share of wage cost for the Icelandic economy is 0.70.

Figure 6.4 shows the relationship between the risk free real rate of interest (RRN_t^s) and the capital-output ratio when it is assumed that there is no risk premium so that the marginal product of capital equals the risk free real rate. The calculations are made for three values on $\widehat{\beta}$: 1) $\widehat{\beta} = 0.64$, 2) $\widehat{\beta} = 0.70$ and 3) $\widehat{\beta} = 0.75$.

This figure shows that if the share of wage cost, $\widehat{\beta} = 0.64$, and the capital-output ratio is just below 12 as it was on average during 2001-2006, the equilibrium risk free real rate of interest is near 4.5%. But if $\widehat{\beta} = 0.70$ this rate is 2.5%, which seems quite low for Icelandic conditions. Assuming that the cost of capital includes a risk premium makes the assumption of the marginal productivity condition still more problematic. For these reasons, the marginal productivity condition for capital is not part of QMM-BG.[22]

[22] It can also be noted that the marginal productivity condition creates difficulties for obtaining an equilibrium solution for the balanced growth version of QMM. The reason for this is that in this case an increase in RRN_t^s leads to a decline in $(K/GDP)_t^s$. At the same time it leads to a decline in regular business investment and in housing investment but as the former effect is stronger, the total effect is that an increase in RRN_t^s decreases the value of exogenous investments

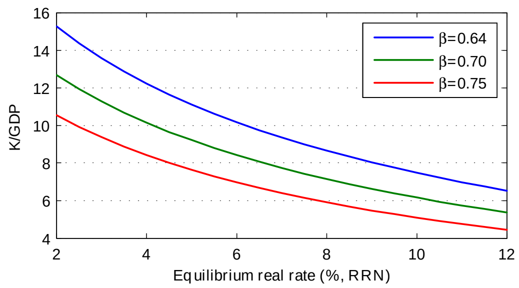

Figure 6.4: Relationship between the capital output ratio (K/GDP) and the equilibrium real rate (RRN)

The equilibrium risk free real rate (RRN) is not directly observable but equations for the monetary policy rate (RS), the long-run nominal rate (RL) and the long-run real rate (RLV) in QMM[23] give that in equilibrium $RRN_t^s = RLV_t^s + \zeta$, where ζ is a parameter which equals 0.00145 given the estimated parameters in QMM. Data on this variable are shown in Figure 6.5 together with data on the (annual) long-run real interest rate with corporate risk premium ($PRBUS_t$) added and the (quarterly) real cost of capital (RCC_t).

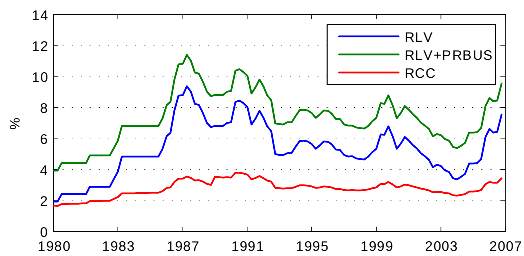

Figure 6.5: Risk free annual real rate (RLV), risk premimum (PRBUS) and quarterly cost of capital (RCC) 1980Q1-2008Q2

The figure shows that during almost all of the last 20 years, RLV_t has been above 4% and sometimes significantly higher. The Icelandic economy has been able to overheat in spite of this relatively high real rate. It therefore seems reasonable to assume that the equilibrium real rate is not below 4%.

If the capital-output ratio is fixed then the share of investment in GDP is also fixed independently of the interest rates. An increase in the equilibrium risk free real rate (RRN_t^s) decreases the equilibrium regular business investments ($IBREG_t^s$) and housing investments (IH_t^s) making the residual investment which is exogenous in QMM, i.e. $IBALU_t^s + IBAIR_t^s + IG_t^s = I_t^s - (IBREG_t^s + IH_t^s)$, larger. Figure 6.6 shows this relationship.

($IBAIR_t^s + IBALU_t^s + IG_t^s$) that is compatible with balanced growth.

[23] i.e. equations (4.1)-(4.7) in Danielsson et al. (2009).

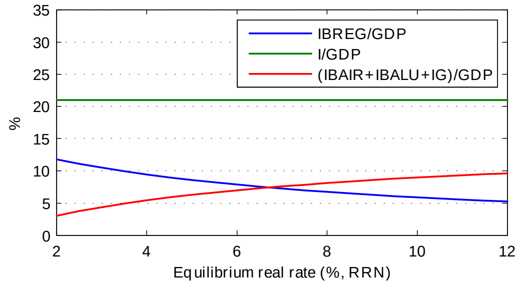

Figure 6.6: Share of regular business investment (IBREG), total investment (I) and exogenous investment (IBAIR+IBALU+IG) in GDP when the equilibrium real rate changes

Figure 6.7 shows the shares of exogenous investment, $IBALU + IBAIR + IG$, in GDP for the period 1980-2007.

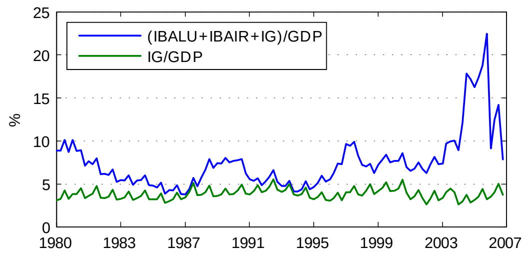

Figure 6.7: Share of exogenous investment in GDP 1980Q1-2007Q4

During most of the period 1980Q1-2007Q4 shown in Figure 6.7 the share of exogenous invesmtent in GDP was between 5% and 10%. The average over the whole period is 7.4% while the average during 1980Q1-2004Q4, i.e. the period up to the very large investment during the last three years, is 6.6%, and the average during the period 1994Q1-2004Q1 is 6.9%. It can be inferred from Figure 6.6 that assuming that exogenous investment is 6% of GDP in equilibrium gives an equilibrium real rate of 4.9%, while if exogenous investmtent is 7% the equilibrium real rate is 6.4%, and if exogenous invesment is 8% equilibrium real rate is 8.0%. This shows that the equilibrium real rate increases quite fast when there is a **permanent** increase in the share of exogenous investment in GDP. A moderate short term increase in exogenous investment requires some tightening of monetary policy and a large increase like the one Iceland experienced during 2004-2007 (see Figure 6.7) requires significant tightening of monetary policy for some years. Unfortunately, the large exogenous investments were not the only factors contributing to the overheating of the economy during this period.

Comparing Figures 6.6 and 6.7 indicates that the equilibrium real rate may be 5% or even higher. Choosing an equilibrium real interest rate of 4.5% is therefore rather conservative.

It was shown above that in most cases it is necessary to adjust the equation for private consumption in QMM to make the model compatible with balanced growth. The size of this adjustment depends on various assumptions, especially the assumption concerning equilibrium capital ouput ratio and equilibrium real rate of interest. It is therefore interesting to see how the size of the adjustment changes when the equilibrium real rate changes. This can be used as a check on the equilibrium real rate. Figure 6.8 shows that the adjustment factor (hpC) increases with the equilibrium real rate.

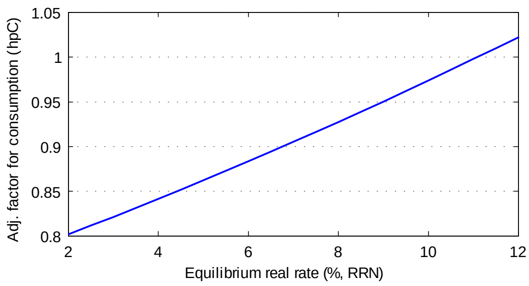

Figure 6.8: The adjustment factor for consumption (hpC) as function of the equlibrium risk free real rate (RRN)

The figure shows that if the equilibrium real rate is below 11.2% then private consumption must be scaled down but if the equilibrium real rate is above 11.2% the private consumption must actually be scaled up compared to what the consumption function in QMM gives so as to meet the requirements of the balanced growth path.[24]

In previous versions of QMM the equilibrium real rate of interest was assumed to be 3%. This is in line with what has been assumed in many similar models for other countries. The arguments above indicate that an upward revision of this variable to 4.5% is warranted. This means that it is assumed that this rate is significantly higher in Iceland than in other countries. The evidence above indicated that the equilibrium real rate in Iceland might be signficantly above 4.5% - or, at least, that it was so until quite recently.

The reasons for this high equilibrium real rate are a bit elusive. As already mentioned the capital-output ratio would suggest that this rate was actually quite a lot lower, even below 3%. On the other hand the population is relatively young compared to other West European nations and even if the population growth has been declining it has been growing at a faster rate than in most West European countries. The GDP in Iceland is volatile, but not extremely so compared to other small open economies, the volatility has been decreasing both in incomes and consumption, but consumption volatility remains significantly larger than income volatility and this difference has been increasing.[25]

[24]The equilibrium value of private consumption must of course decrease when public consumption increases and vice versa.

[25]See Daníelsson (2008) which discusses the Great Moderation in the variation in Icelandic data

7. Sensitivity of equilibrium values to exogenous variables

The equilibrium solution to QMM_BG, calibrated as explained above, gives a sustainable share of consumption in nominal GDP of 52.9%, well below 58.0%, which is the average for the sample period used for estimating the consumption function in QMM, 1994Q1-2006Q4. The assumed capital-output ratio gives investment as a share in nominal GDP of 21.6% and the assumed share of public consumption in real GDP, 23.6%, which is equal to the average during 1994Q1-2006Q4, and the equilibrium relative price of public consumption, give the equilibrium public consumption of 24.0% of nominal GDP. The equilibrium share of the trade balance is 1.4% of nominal GDP, well above the average trade balance during 1994Q1-2006Q4 of -3.5% and also well above the average trade balance during 1980Q1-2006Q4 of -1.6%.

Table 7.1 below shows the equilibrium values for some important relative prices and exchange rates.

Table 7.1: Actual and equilibrium values for some endogenous variables

	2006			2008Q2	
Variables	Actuals	Eq. val.	Difference	Actuals	Eq. val
Exch. rate index‡	121.5	152.0	25.2%	152.9	166.2
EER	1.078	1.349	25.2%	1.357	1.474
REX	1.071	0.830	−22.6%	0.934	0.830
WCPI	1.137	1.137	0.0%	1.205	1.205
PGDP	1.326	1.326	0.0%	1.536	1.536
CPI	1.309	1.272	−2.8%	1.526	1.473
W	1.535	1.414	−7.9%		
Real wage	1.173	1.112	−5.2%		
Private cons.	530741	450556	−15.1%		

Notes: ‡ Official exchange rate index published by the Central Bank of Iceland

The equilibrium values in the second column of Table 7.1 are calculated using the domestic price level ($PGDP$) and foreign price level ($WCPI$) in 2006 and other assumptions as explained in Tables 6.1 and 6.2 in Section 6 above. The table shows that to reach equilibrium, the real exchange rate would have had to be 23% below the actual value, the nominal exchange rate index would have to be 25% higher, the wage level 8% lower, the CPI level 3% lower and the real wage level 5% lower than it actually was in 2006.[26] The second last column in Table 7.1 shows actual values of the same variables in 2008Q2. The last column shows the equilibrium value for the real exchange rate (REX), which is also given in the second column, and the equilibrium values for EER and CPI, based on the actual values on $WCPI$ and $PGDP$ in 2008Q2 and the equilibrium values for REX and $CPI/PGDP$. The values for the nominal exchange rate (EER) and the official exchange rate index are

on the real economy until 2007. The period after 2007 has seen large increases in volatility both in Iceland and in most other countries.

[26] R. Tchaidze (2007) uses three different methods to estimate the equilibrium real exchange rate for Iceland. The estimates range from 8-23% below the average rate during 2006. Sighvatsson (2000) discusses the equilibrium real exchange rate of the Icelandic krona.

then calculated using the relevant formulas and the actual values for the exogenous variables in 2008Q2.

Figure 7.1 below shows how the equilibrium values change when the value of some exogenous variable changes compared to the value given in Tables 6.1 and 6.2 above. In the case of the exchange rates, the results are shown by measuring the relative difference between the equilibrium values and actual values in 2006 but figures showing shares of economic aggregates in GDP show the equilibrium values. In all cases it is assumed that the equilibrium real rate of interest is 4.5% and the capital-output ratio 11.7.

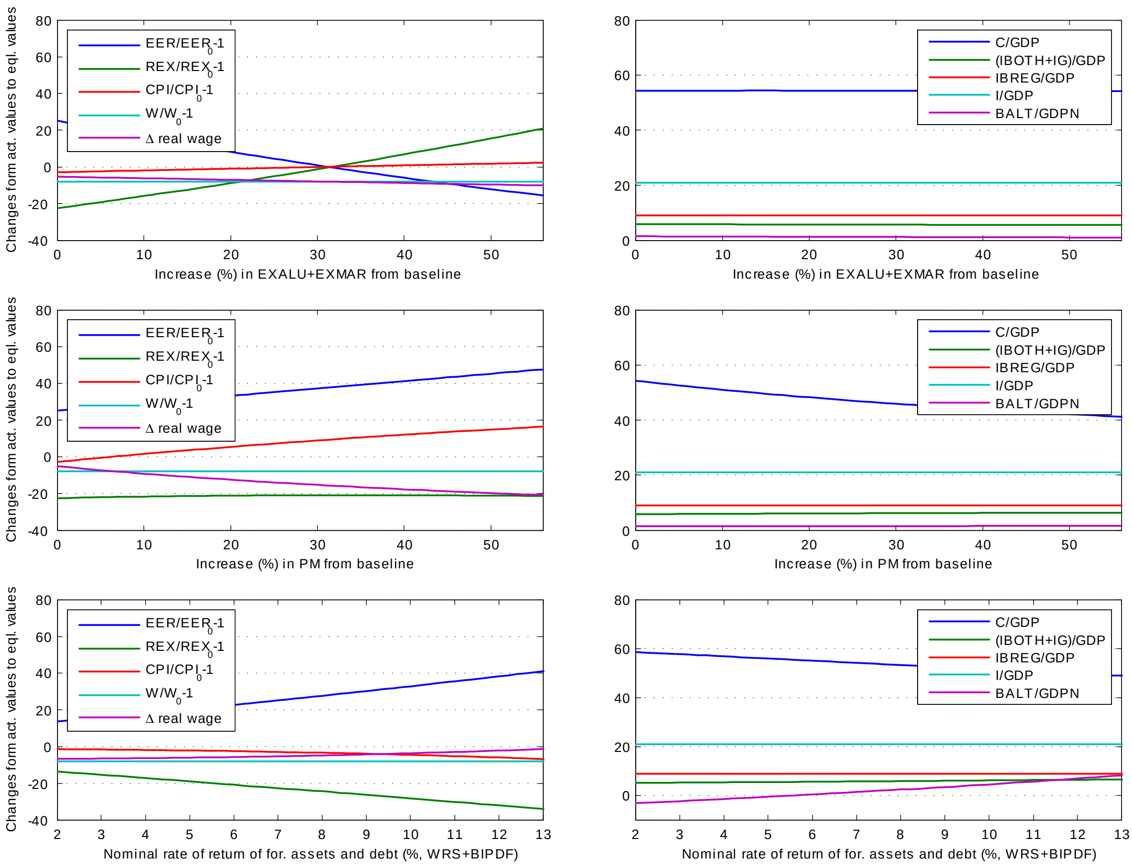

Figure 7.1: Deviations caused by 1% higher policy rate (RS) during 4 quarters

The figures in the first row in Figure 7.1 show how some equilibrium values change if the assumption concerning the total volume of export of aluminium and marine products ($EXALU + EXMAR$) is changed, compared to its long-run value, as indicated on the horizontal axis. The figures in the second row show how equilibrium values change when assumptions concerning the relative price of import (PM) are changed and the figures in the third row show how these values change when assumptions concerning the rate of interest on foreign debt ($WRS + BIPDF$)[27] are changed.

[27] Assumed here to be equal to the rate of return on foreign assets hold by Icelanders and equal

Let us finally consider what happens in QMM-BG if labour productivity increases but everything else remains the same including the exogenously given export of aluminium and marine products. The results are shown in Figure 7.2.

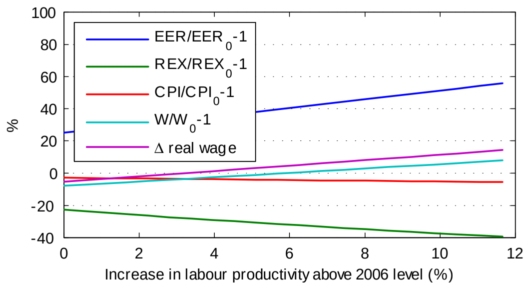

Figure 7.2: Differences between actual values in 2006 and equlibrium values in selected variables when labour productivity is increased

The figure shows that in this case the wage rate and the real wage decrease less or even increase if the increase in labour productivity is sufficiently large. To redirect the increased demand towards domestic products to maintain equilibrium in the external balance, the real exchange rate has to depreciate more than it would have to do without this increase in labour productivtity.

8. Impulse response functions from QMM and QMM-BG

By construction, QMM-BG simulates points on the balanced growth path if it starts from an arbitrary point on such a path. As explained in the introduction, impulse response functions from the forward looking version of QMM-BG requires some further analysis. The discussion below will therefore focus on the backward looking version of QMM-BG, which is derived from the backward looking equations in version 2.0 of QMM. This version of QMM-BG is stable in all respect except one. Even slight disturbance from a constant growth path will send the system on a path to indefinite accumulation of net foreign debt or net foreign assets. Figure 8.1 shows results from simulations where the policy rate (RS) has been increased from the equilibrium level of 7% to 8% during the first four quarters, and after that a backward looking Taylor rule determines the policy rate. The panel on the left in Figure 8.1 shows the deviations of the policy rate, output gap ($GAPAV$) and the rate of inflation (INF) from their equilibrium values (indicated with the superscript *) on the balanced growth path. The figure shows that these variables return to their equilibrium values after some 30-40 periods while the real exchange rate (REX) and the nominal exchange rate (EER) either do not return to their equilibrium values in the long-run or do so extremely slowly. In the case shown in Figure 8.1 the real exchange rate and the

to the rate of return on assets in Iceland hold by foreign nationals. As noted above the basic assumption is that this rate of interest is 7%.

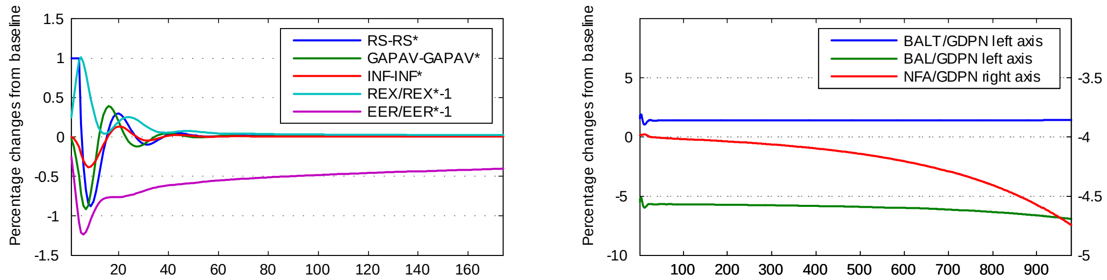

Figure 8.1: Simulation with QMM_BG with the backward looking equation for the real exchange rate in QMM

nominal exchange rate seem to be returning to their equilibrium values, but when they do, the additional external debt accumulated during the time when the real exchange rate was above its equilibrium requires a new and lower real exchange rate to restore equilibrium where the debt level stabilises.

The panel on the right in Figure 8.1 below shows the ratios of the balance of trade ($BALT$), the current account (BAL) and the net foreign assets (NFA) to nominal GDP. The figure shows that even this slight disturbance of the equilibrium pushes the economy onto a path where the current account and the net foreign assets increase indefinitely as a share of nominal GDP.

The reason for this anomalie is that in the version of QMM-BG, which uses the backward looking version of the equations in QMM, there is no feedback mechanism preventing the economy from increasing its debt indefinitely. Such mechanisms are at work in capitalist economies and in times of financial distress it is learned that they are highly non-linear and contain large stochastic terms. Because of limited knowledge of the true mechanisms, and, of course, for the sake of simplicity, the linear term $\phi \times [NFA_{t-1}/GDPN_{t-1} - \psi]$ is added to the equation for the real exchange rate in QMM, giving the equation:[28]

$$rex_t = rex^* + \gamma\,(rex_{t-1} - rex^*) + RID_t + \phi \times [NFA_{t-1}/GDPN_{t-1} - \psi] \qquad (8.1)$$

where γ is a parameter estimated to be 0.946 and

$$RID_t = \ [(RS_t - \Delta_4 cpi_t) - (WRS_t - \Delta_4 wcpi_t) - RISK_t]\,/4$$

is the risk-adjusted real interest rate differential between Iceland and abroad,[29] ψ is the ratio $NFA_t/GDPN_t$ used for constructing the balanced growth path and ϕ is an adjustment parameter. Calibrating ϕ to 0.007 gives reasonable adjustment paths as shown below.

[28] This formulation ensures that not only will the real exchange rate return to its equilibrium but also the ratio of net foreign assets and nominal GDP.

[29] The division by 4 stems from the fact that the interest rate differential is measured as an annual rate, whereas the real exchange rate is quarterly.

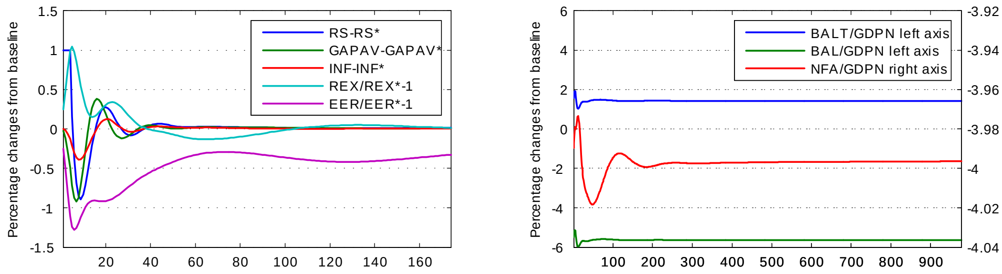

Figure 8.2: Simulation with QMM_BG with the revised equation for the real exchange rate

Figure 8.2 shows the outcomes of a simulation using the same assumptions and the same model as in the simulation shown in Figure 8.1, except for the formula for the real exchange rate.

Here it is only the nominal exchange rate that does not return to the equilibrium value in the left panels of Figures 8.1 and 8.2. A longer simulation shows that it eventually returns to the equilibrium value but the convergence is very slow. The right panel in Figure 8.2 shows that the real exhange rate equation (8.1) ensures that the ratios of the current account and that of the net foreign assets to nominal GDP return to their equilibrium values in the long-run. For this to happen the real exchange rate must be below the equilibrium value for a while so that the current account improves and the external debt returns to the original level.

Figure 8.3 shows the differences between the simulation shown in Figure 8.1 and the simulation shown in Figure 8.2. The superscript S indicates that the REX-equation (8.1) above was used in the simulation. The figure shows the differences between this simulation with QMM-BG and a simulation using the backward-looking equation in QMM for REX.

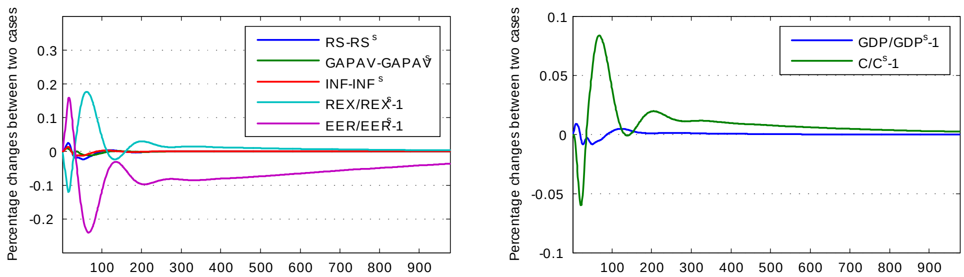

Figure 8.3: Difference between simulations shown in Figures 8.1 and 8.2

Impulse-response functions depend on the initial conditions as well as on the model itself. In Figure 8.4 two sets of impulse-responses are shown. In both cases a base simulation has been made and then another simulation where the rate of interest is 1% higher than in the first one during the first four quarters before the Taylor rule takes over. The figures show the differences between the two runs. In the first set of

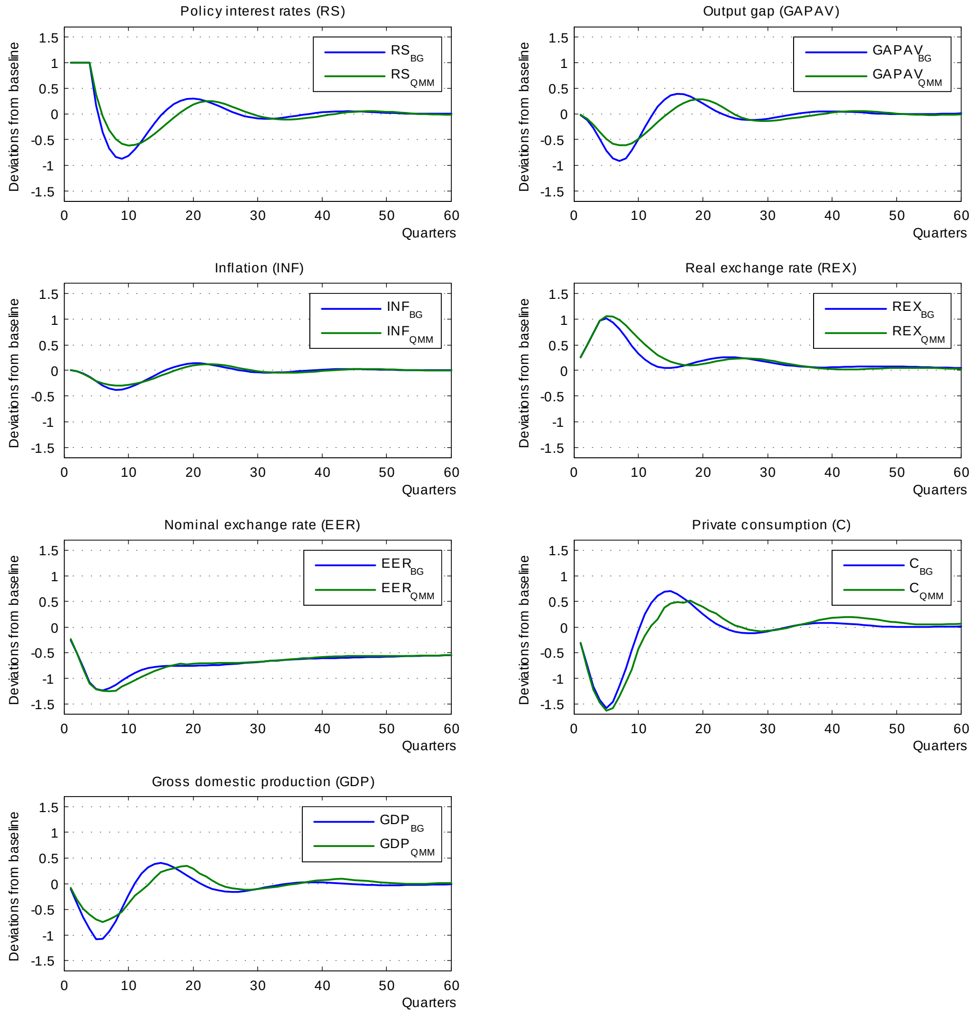

Figure 8.4: Outputs from simulations with QMM and QMM_BG

simulations, indicated with "-BG", this simulation is done with QMM-BG and the base simulation is the equilibrium balanced growth path. In the second set, indicated with "-QMM", the base simulation is a simulation with QMM starting in 2008Q3 using the Taylor rule for the policy rate while the second simulation starts in the same period, the policy rate is set 1% higher than in the base simulation during the first four quarters and after that the Taylor rule is used. In both cases, the equation for the real exchange rate is the backward-looking one in QMM without any increase in the risk premium when net debt increases.

The figures show that the differences between the outcomes from the two simulations are small in most cases. The largest differences are in the case of GDP and the output gap where the effects from an RS-shock are somewhat larger when the shock is applied to QMM-BG than when it is applied to QMM. Both the nominal and the real exchange rate return to their former equilibrium values eventually, but the process of adjustments is very slow.

9. Conclusions

In this paper it is concluded that starting from a given point in Iceland's recent history and assuming realistic paths for future developments of exogenous variables, QMM does not converge to a balanced growth path but accumulates ever-increasing external deficits. It is argued that, given a rate of investment determined by some technologically given capital output ratio, the main reason for the unsustainable paths generated by QMM in the long-run is unsustainably high level of private and public consumption. To construct a model which is consistent with a balanced growth path it is therefore necessary to make some adjustments that affect the level of consumption. The paper describes the construction of such a model, QMM-BG, which is consistent with a given growth rate and given reasonable path for the exogenous variables. To affect the dynamic properties of individual equtions as little as possible, the adjustments made to individual equations in QMM consisted mainly of changing constant terms that determine the long-run levels of the variables. When constructing QMM-BG similar adjustments were made to the equation for the demand for housing investments as were made to the equation for private consumptions. The real disposable income was also adjusted to what is required for maintaining the given ratio of household net financial wealth to disposable income.

The paper discusses calibration of some exogenous variables in QMM, especially the risk free equilibrium real rate of interest and the share of wage cost. The calibrated version of QMM-BG was used to estimate equlibrium real exchange rate for the Icelandic economy. The result was that it is 17% below the average value in 2000 and nearly 23% below the average value in 2006. This value is on par with the lowest values for this variable since 1980 and until the collapse of the banking system in Iceland in October 2008. Until then, the lowest real exchange rate was in 2001Q4 when it was 17% below the average in 2000.

The paper compares the dynamic properties of QMM-BG and QMM and concludes that for the backward version of the model there is no significant differences in the impulse-resoponses between the two models. Comparisons of the dynamic properties of the forward versions of the two models requires some further work.

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