Endogenous Risk

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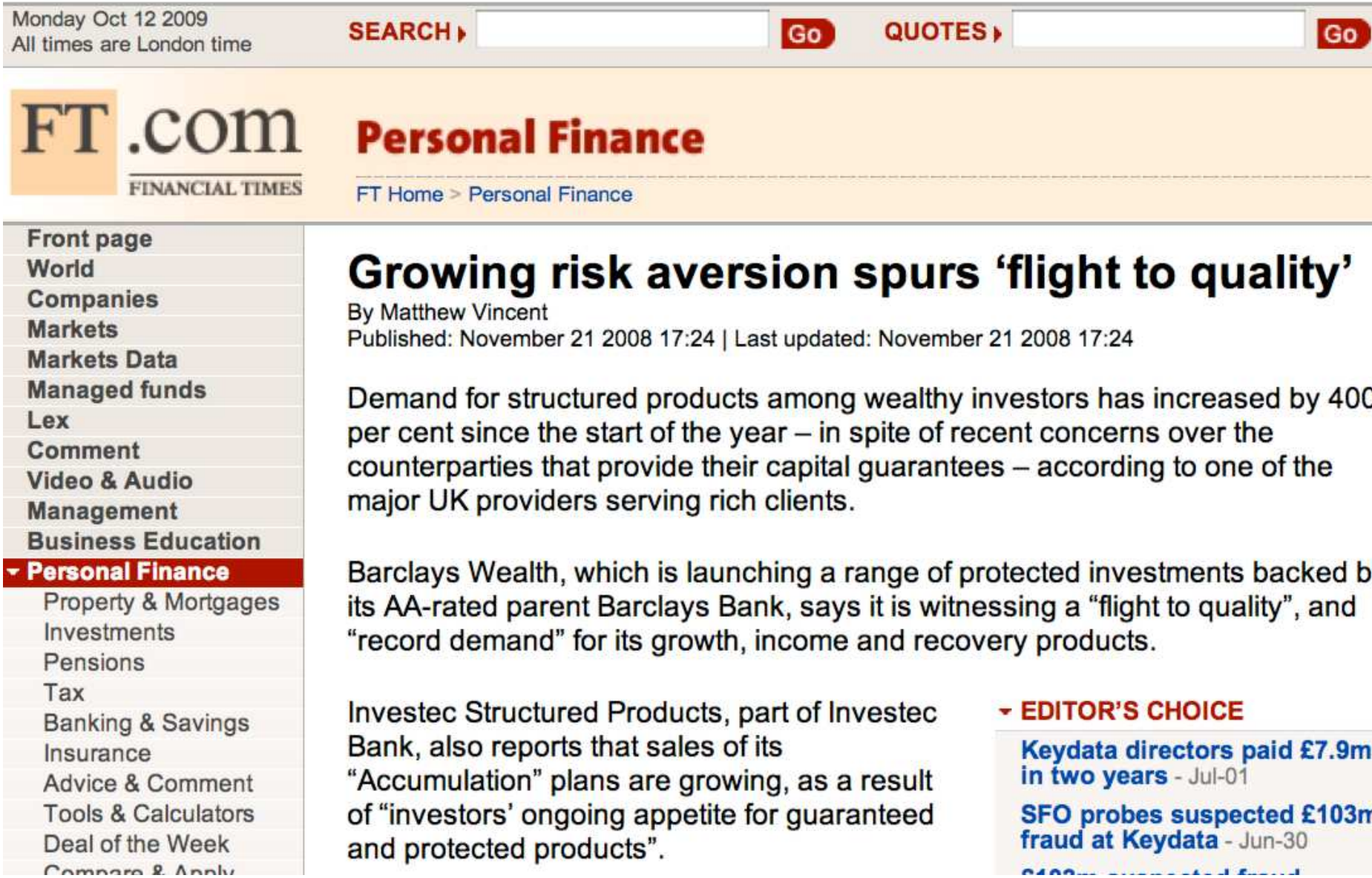
Talk is based on 3 Papers

- Risk Appetite and Endogenous Risk, `http://risk.lse.ac.uk/rr/files/JD-HS-JZ-34.pdf`
- Modelling Financial Turmoil Through Endogenous Risk, `http://www.voxeu.org/index.php?q=node/3243`
- Endogenous Risk and Price Dynamics (in progress)

Large price movements

- Exogenous single event — e.g. macro announcement
- Endogenous responses
  - Perhaps speculative price buildup
  - Followed by rapid price drops
  - "*Up the stairs — down the elevator*"
- Commentators say "risk aversion went up"
- But risk aversion is preference parameter



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Growing risk aversion spurs 'flight to quality'

By Matthew Vincent
Published: November 21 2008 17:24 | Last updated: November 21 2008 17:24

Demand for structured products among wealthy investors has increased by 400 per cent since the start of the year – in spite of recent concerns over the counterparties that provide their capital guarantees – according to one of the major UK providers serving rich clients.

Barclays Wealth, which is launching a range of protected investments backed b its AA-rated parent Barclays Bank, says it is witnessing a "flight to quality", and "record demand" for its growth, income and recovery products.

Investec Structured Products, part of Investec Bank, also reports that sales of its "Accumulation" plans are growing, as a result of "investors' ongoing appetite for guaranteed and protected products".

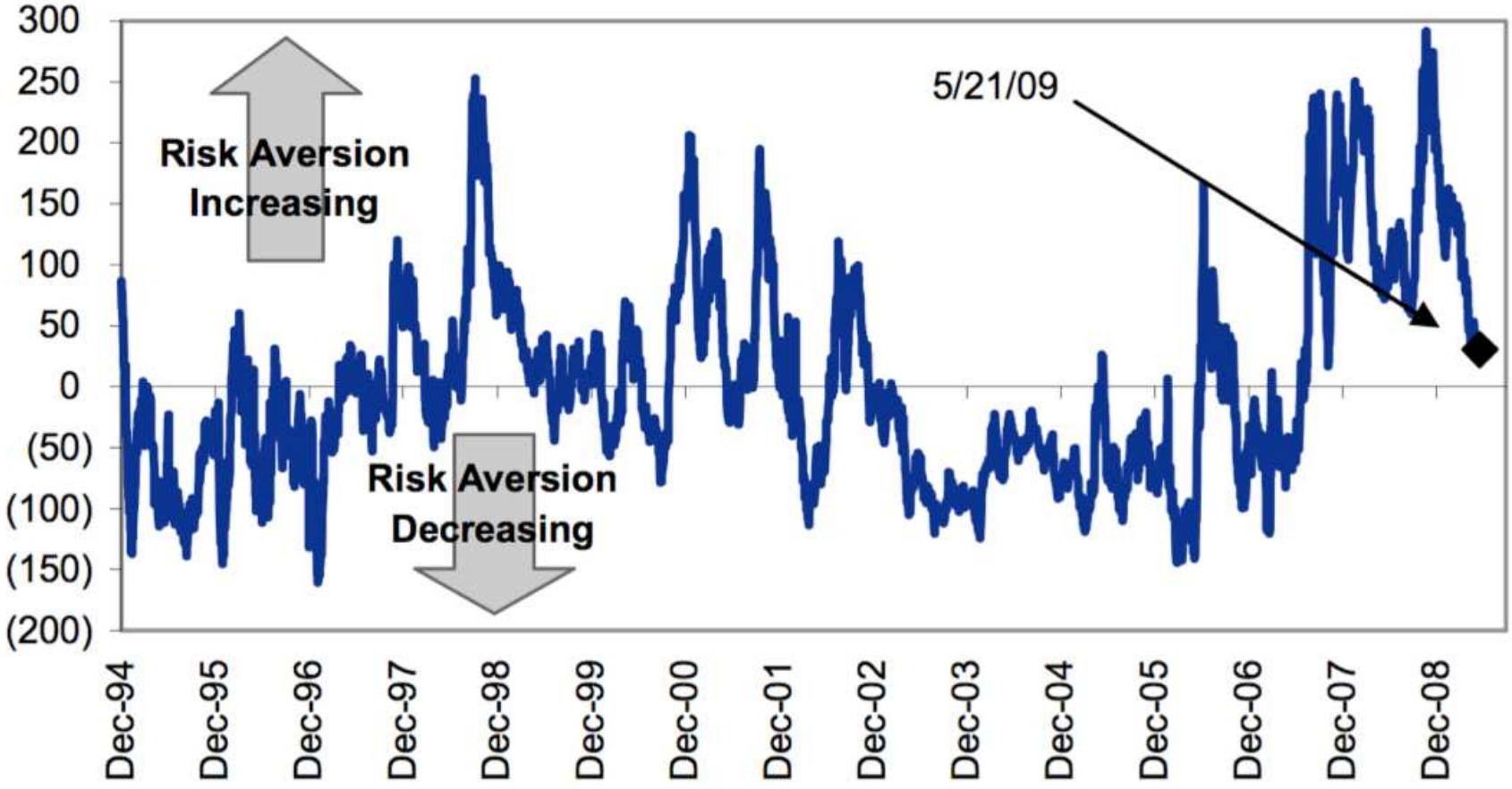
EDITOR'S CHOICE

Keydata directors paid £7.9m in two years - Jul-01

SFO probes suspected £103m fraud at Keydata - Jun-30

Exhibit 35: Goldman Sachs Risk Barometer

Index is number of standard deviations from the average * 100

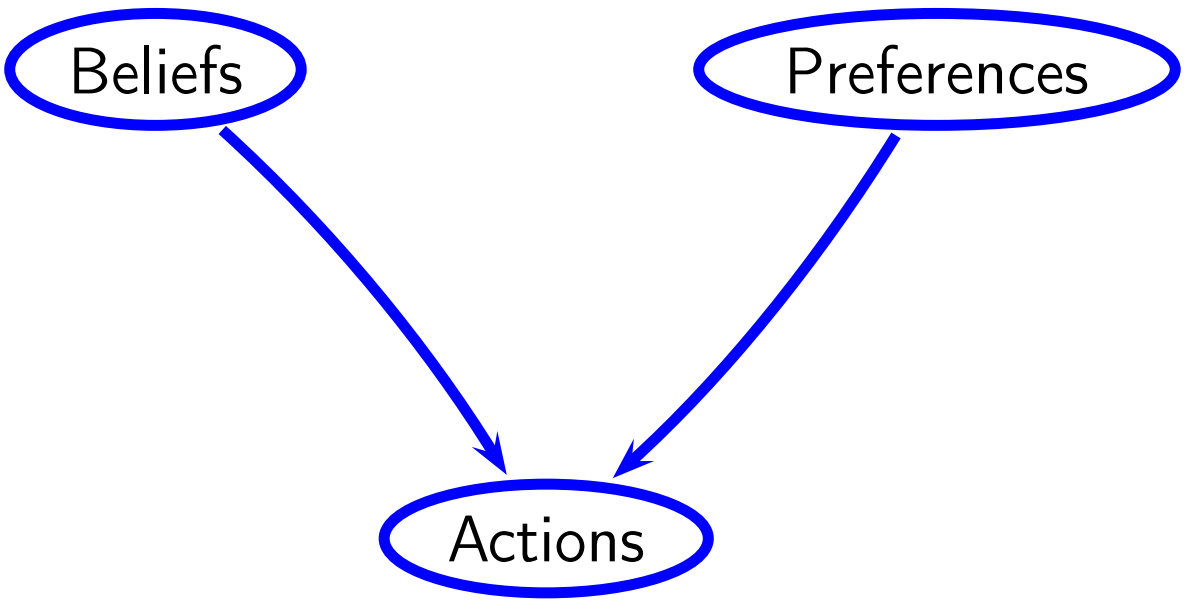


Note: metrics included are implied option volatility (S&P 500 and Nasdaq 100), normalized skew, high yield credit spreads, credit derivative swap spreads, mutual fund net flows, and cash levels.

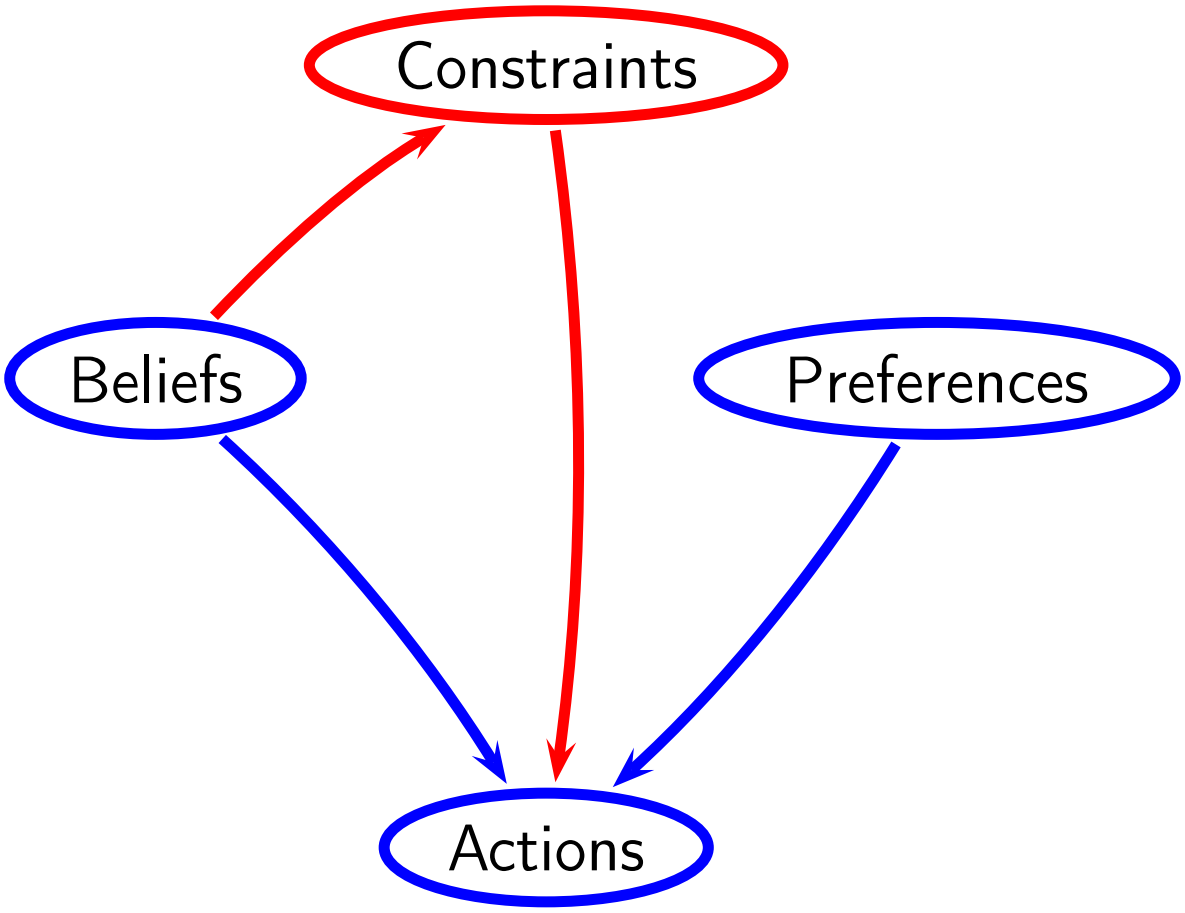
Source: Goldman Sachs Research.

- How can (apparent) preferences and beliefs be linked?
- ... through *constraints* faced by traders
- Value-at-Risk (VaR) constraints bind harder in more volatile conditions
- Past outcomes influence both
  - **(i)** how tight constraints bind (apparent preferences) and
  - **(ii)** current forecasts of future outcomes
- Upshot is that traders appear to become more risk averse when faced with greater uncertainty
- Then there is potential for feedback

Beliefs and Preferences

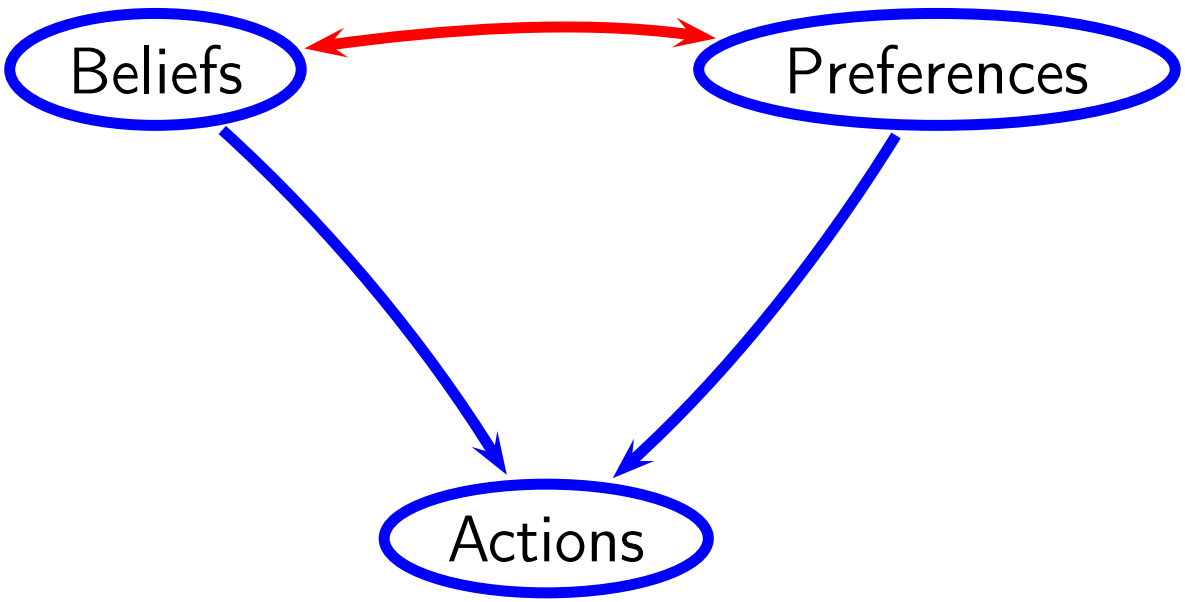


Beliefs and Preferences



Beliefs and Preferences

To outside observer,
beliefs and preferences
appear to be linked



Keynesian beauty contest

"It is not a case of choosing those [faces] which, to the best of ones judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

Keynes, General Theory of Employment Interest and Money, 1936.

Endogenous risks vs. Exogenous risks

- *Endogenous risk*: the risk from shocks that are generated and amplified *within* the system
- *Exogenous risk*: shocks that arrive from *outside* the system
- Analogies
  - A financial hedge (futures contract) vs. a weather hedge (umbrella)
  - Poker vs. Roulette
- Essentially situations where an agent affects outcomes
  vs.
  situations where the agent cannot

Millennium Bridge

- First new Thames crossing for over a hundred years
  - New design, extensive tests, riskless
  - Opened by the Queen on June 10th 2000
- What happened?

Millennium Bridge

- First new Thames crossing for over a hundred years
  - New design, extensive tests, riskless
  - Opened by the Queen on June 10th 2000
- What happened?
  - Wobbled violently within moments of bridge opening
  - Remain closed for the next 18 months

Millennium Bridge

- New design
- Tested with extensive simulations
- All angles covered
- No endogenous shocks
- Riskless

What Endogeneity?

- Pedestrians had some problems
- Bridge closed

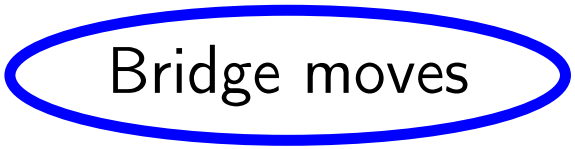
What happened?

- Took the engineers some time time to discover what happened

What went wrong?

- An engineering answer
  - Cause: horizontal vibrations at 1 hertz
  - Walking pace: 2 steps per second, i.e. 2hertz
  - Producing 1 hertz horizonal force
- Why should it matter?
  - People's sway to the left and right cancel out each other
  - Only a problem when people walked in step
  - Probability of a thousand people walking at random ending up walking exactly in step? - *close to zero*
  - If individual steps are independent events, but...

Given feedback...near certainty!



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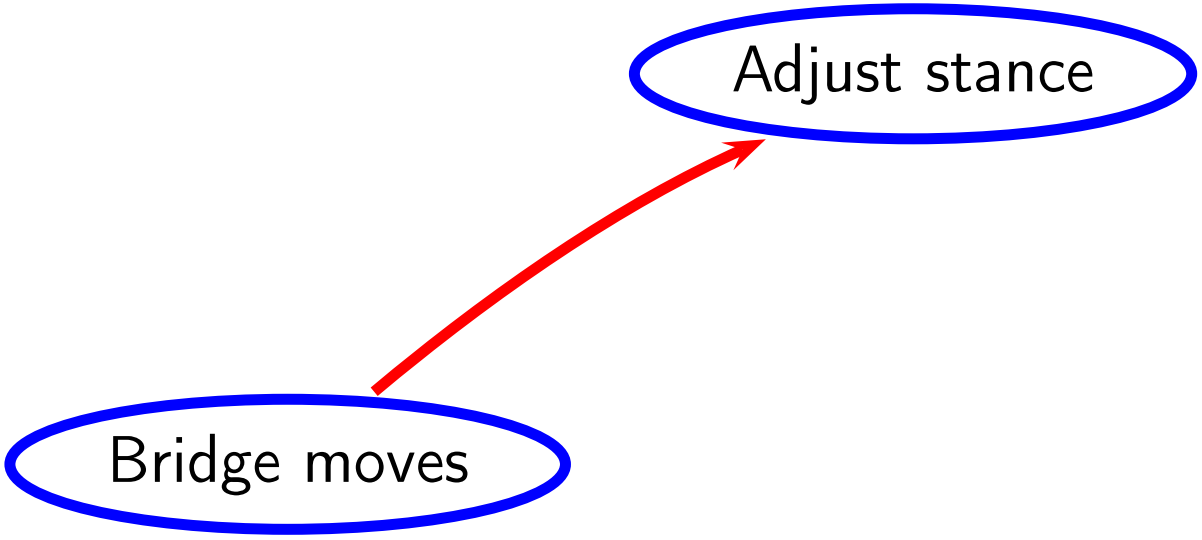

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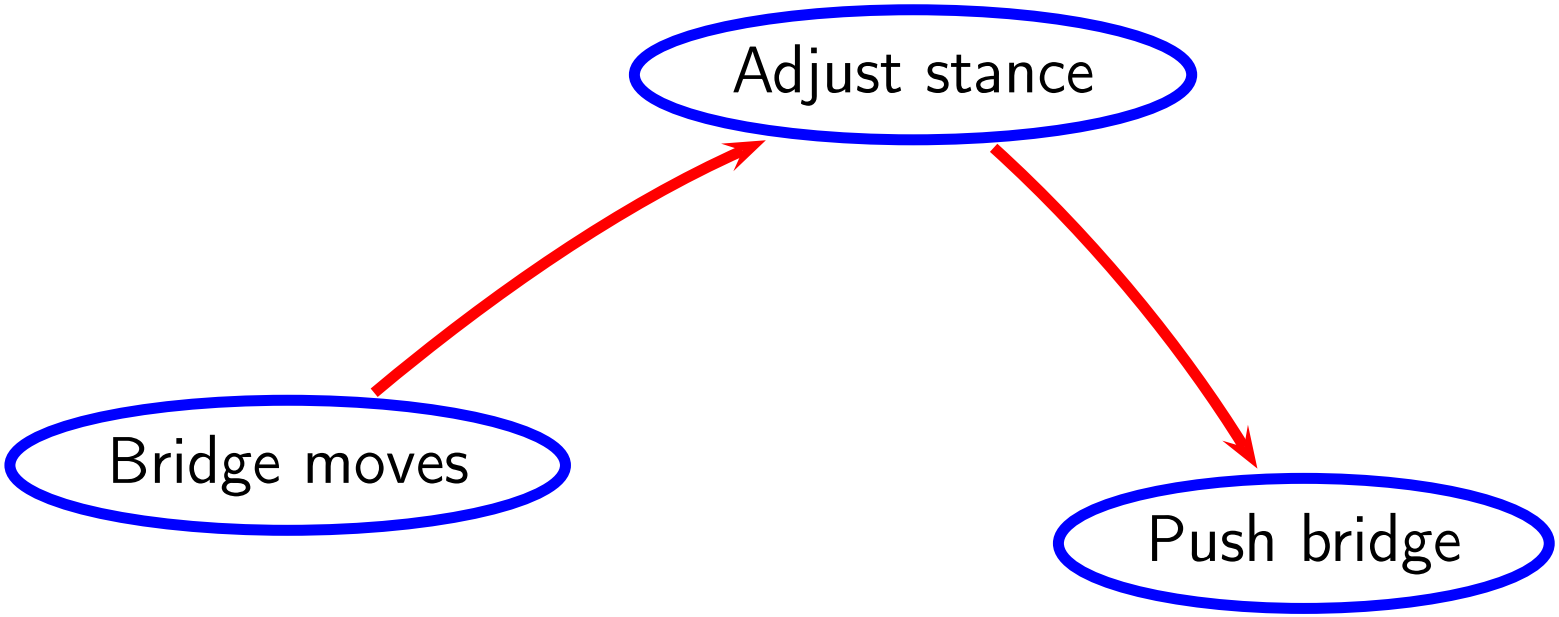

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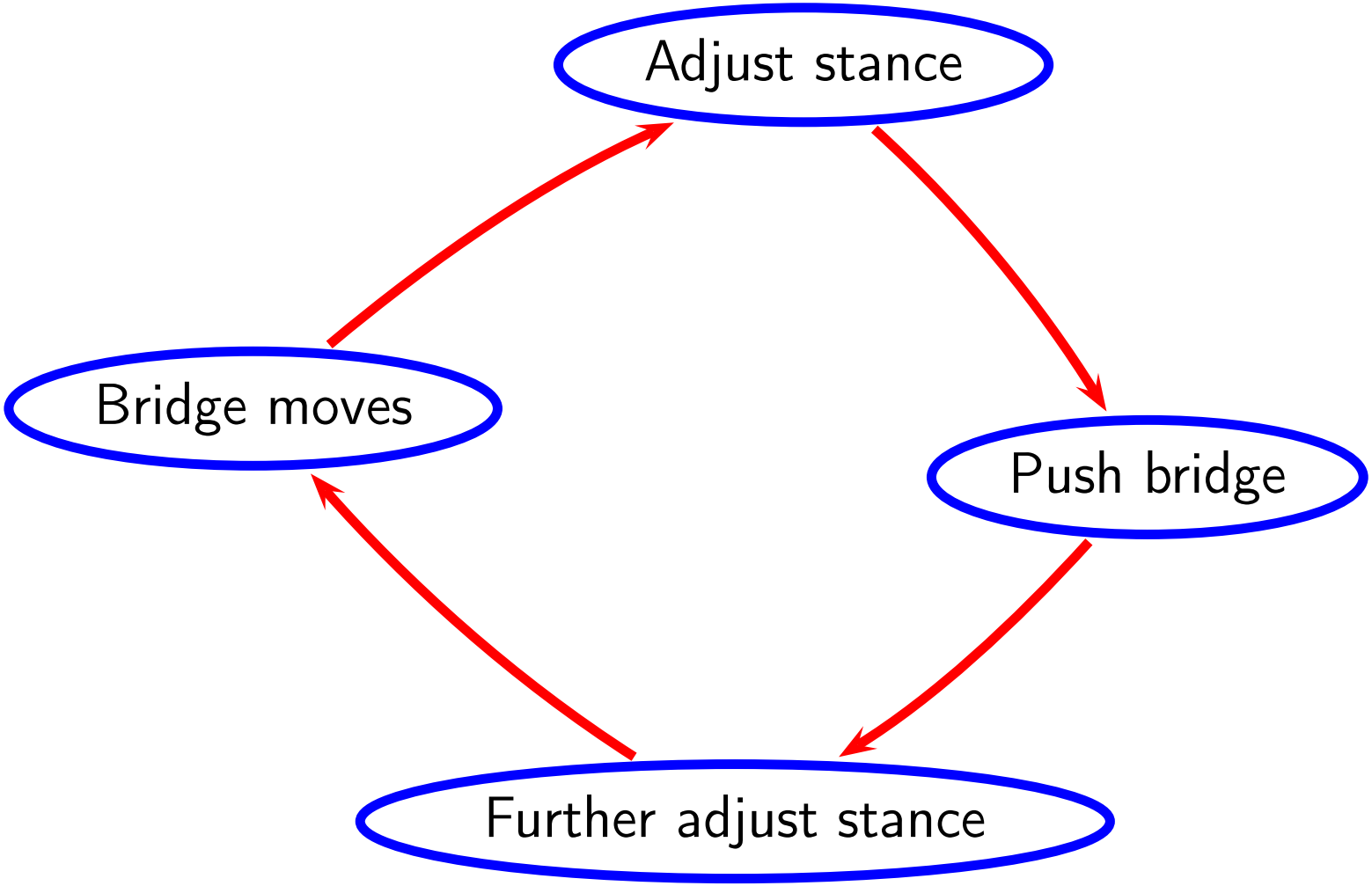


Model

Endogeneity of Risk

- True risks impacting financial markets are attributable (at least in part) to actions of market participants.
- Endogenous risk is fixed point of mapping:

$$\text{perceived risk} \Rightarrow \text{actual risk}$$

- Solve for *stochastic volatility* function $\sigma(V)$ in closed form, where
  - V is *capital* of active traders
  - Fundamental uncertainty is constant (iid)

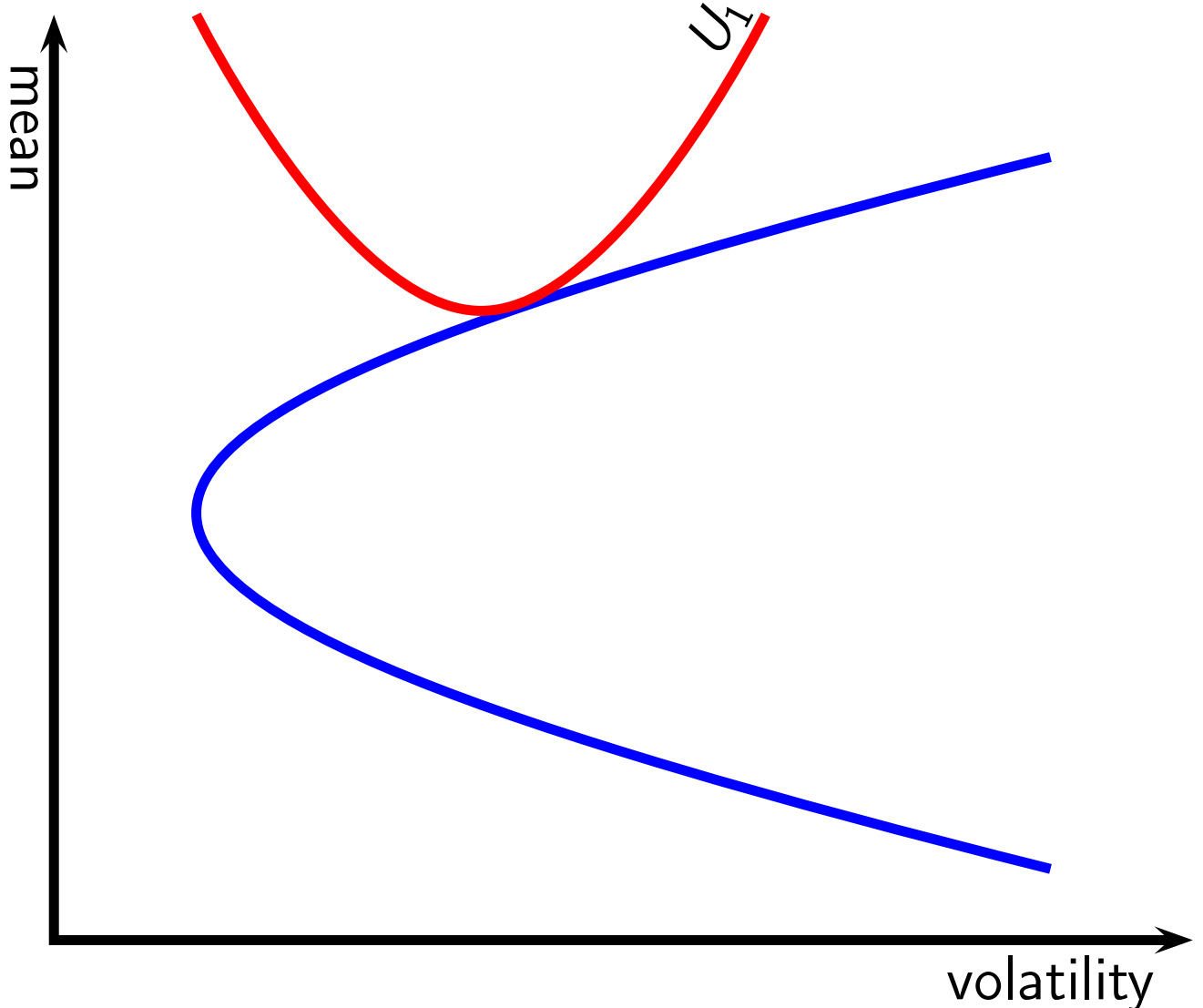
Some Themes

- Risk appetite $\neq$ risk aversion
  - *Risk neutral* traders with *Value–at–Risk* (VaR) constraints
  - Risk appetite depends on how hard VaR constraint *binds*, i.e. on forecasts (beliefs) and trading capital
  - *Lagrange multiplier* enters like a risk aversion parameter
  - "As if" preferences shift with beliefs and losses/gains
- Distinction is more than semantic quibble
  - Risk-bearing capacity depends on recent outcomes (and seemingly evaporates during crises)
  - Crisis is not just about "jumps", but about *amplification of distress*

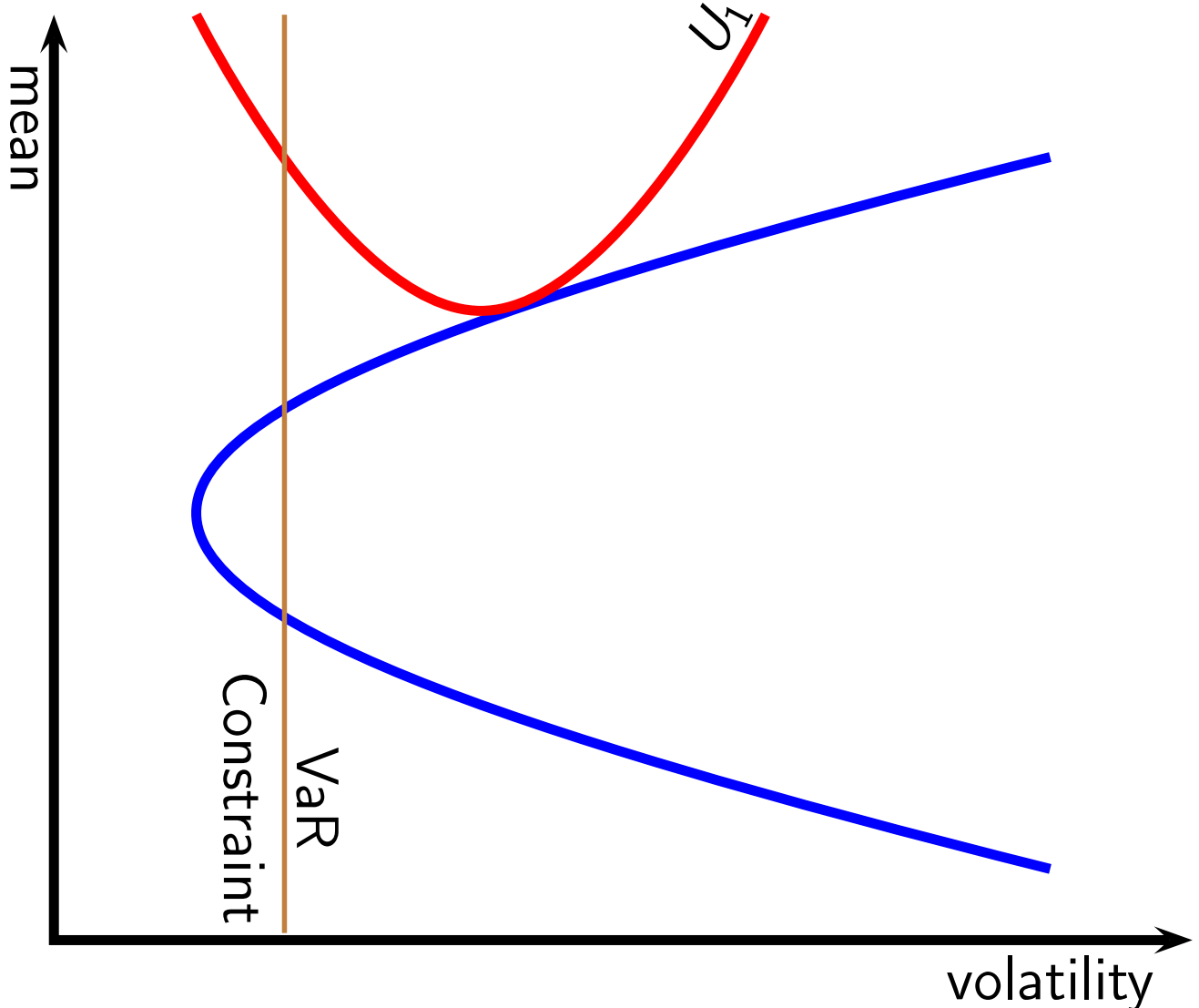
More Themes

- *Endogenous correlation* of returns
  - Returns on risky assets are correlated even though fundamental shocks are independent
- Financial regulation (e.g. Basel II) built on individually optimal risk management is flawed

Mean–Variance with VaR constraints



Mean–Variance with VaR constraints



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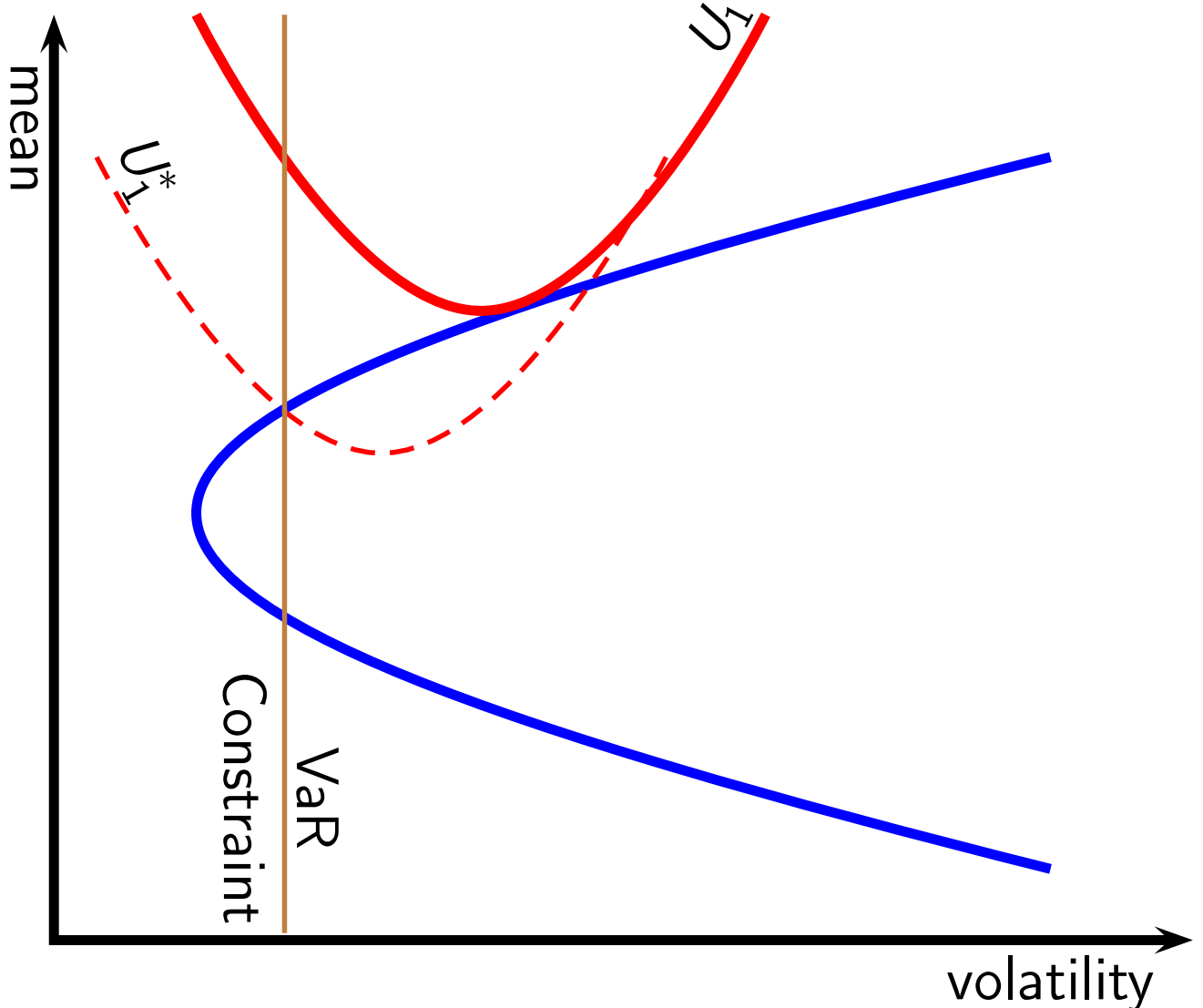

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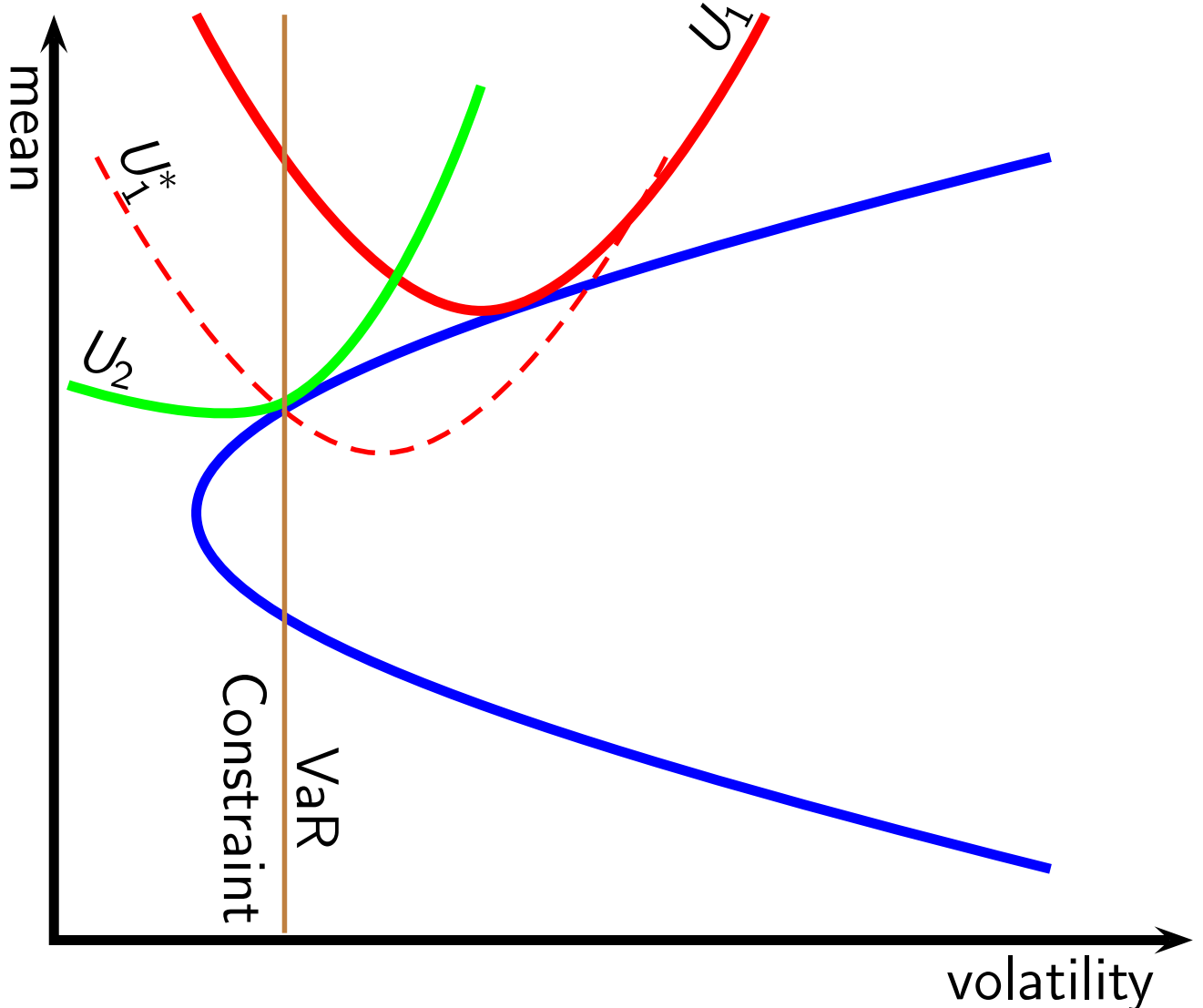


Model

- Two types of traders
  - Active traders (risk-neutral traders with VaR constraints)
  - Passive traders (residual demand/supply curves that close the model)

Solve for rational expectations equilibrium (REE) with respect to active traders' beliefs

Portfolio Choice of Active Traders

maximization problem is

$$\max_{D_t} \; rV_t + D_t^\top(\mu_t - r) \qquad \text{subject to} \qquad \alpha\sqrt{D_t^\top \sigma_t \sigma_t^\top D_t} \leq V_t$$

First-order condition

$$\mu_t - r = \alpha(D_t^\top \Sigma_t D_t)^{-1/2}\gamma_t \Sigma_t D_t$$

where γ_t is Lagrange multiplier for VaR constraint,

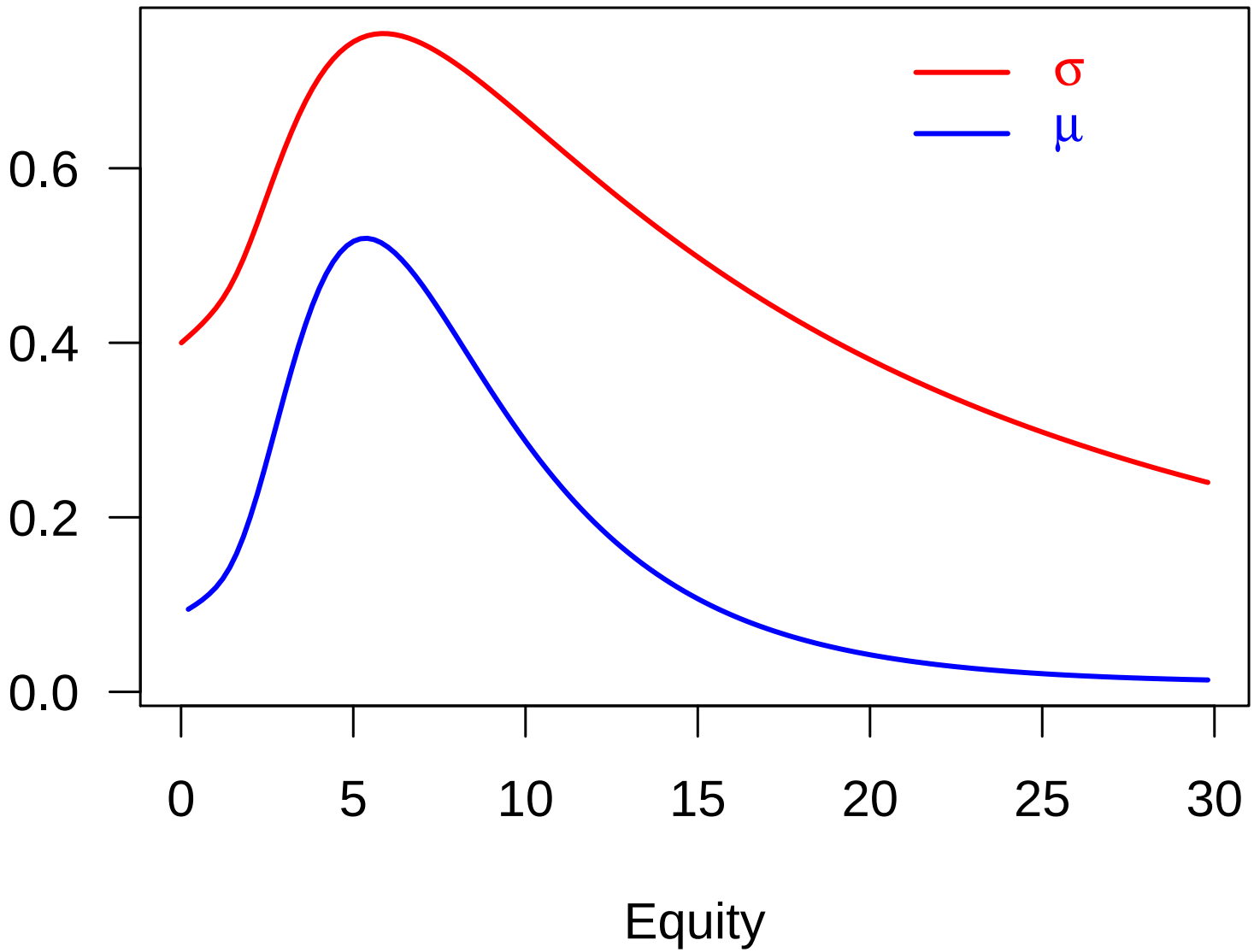
"As if" Preferences

Optimal portfolio is similar to mean-variance optimal portfolio where the Lagrange multiplier γ_t appears like a risk-aversion coefficient and is

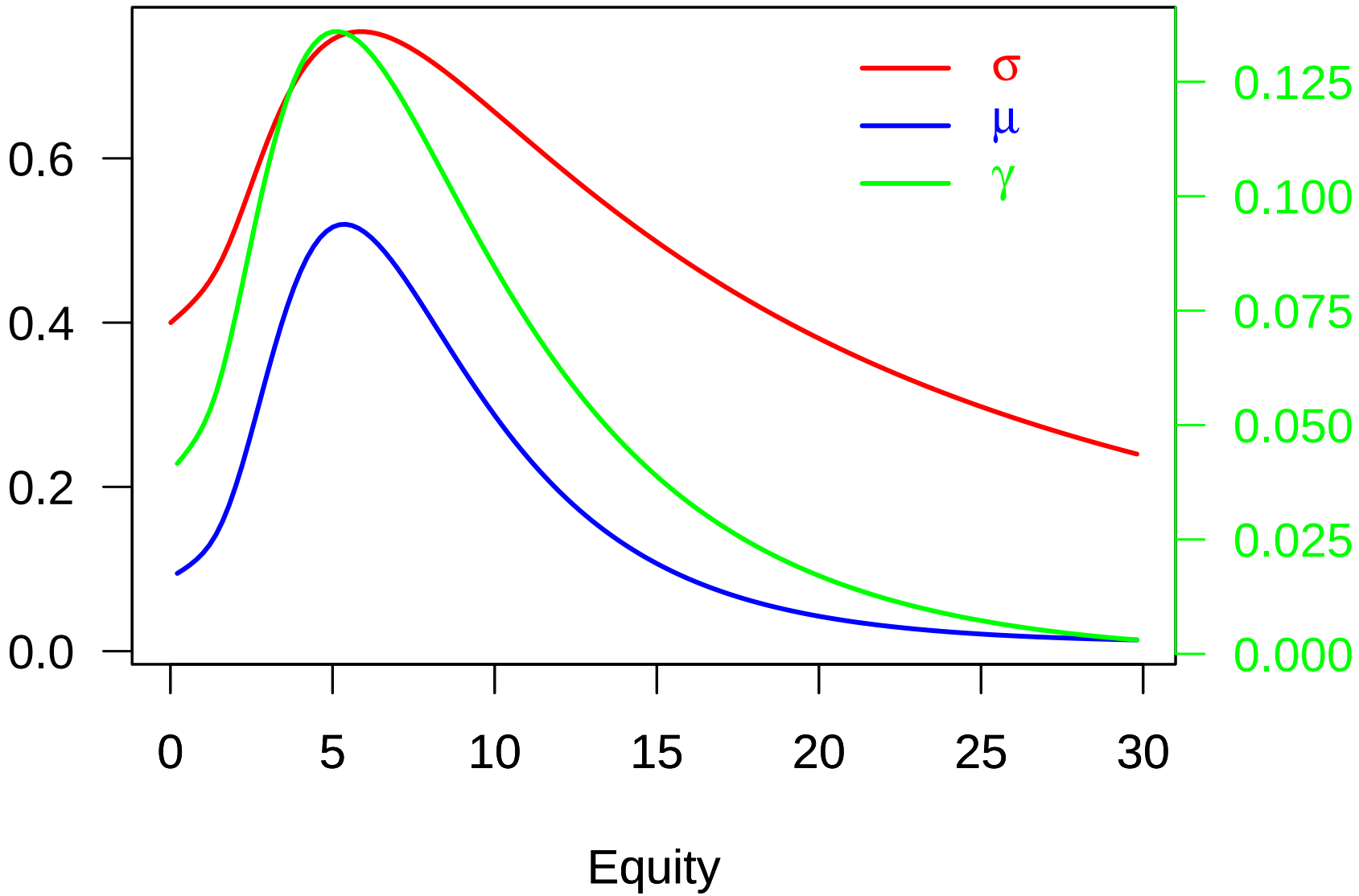
- proportional to generalized Sharpe ratio $\sqrt{\xi}$
- does not depend directly on equity V_t

Interpretation. Additional unit of capital relaxes VaR constraint by multiple α of standard deviation, raising expected return by risk-premium on the portfolio per unit of standard deviation

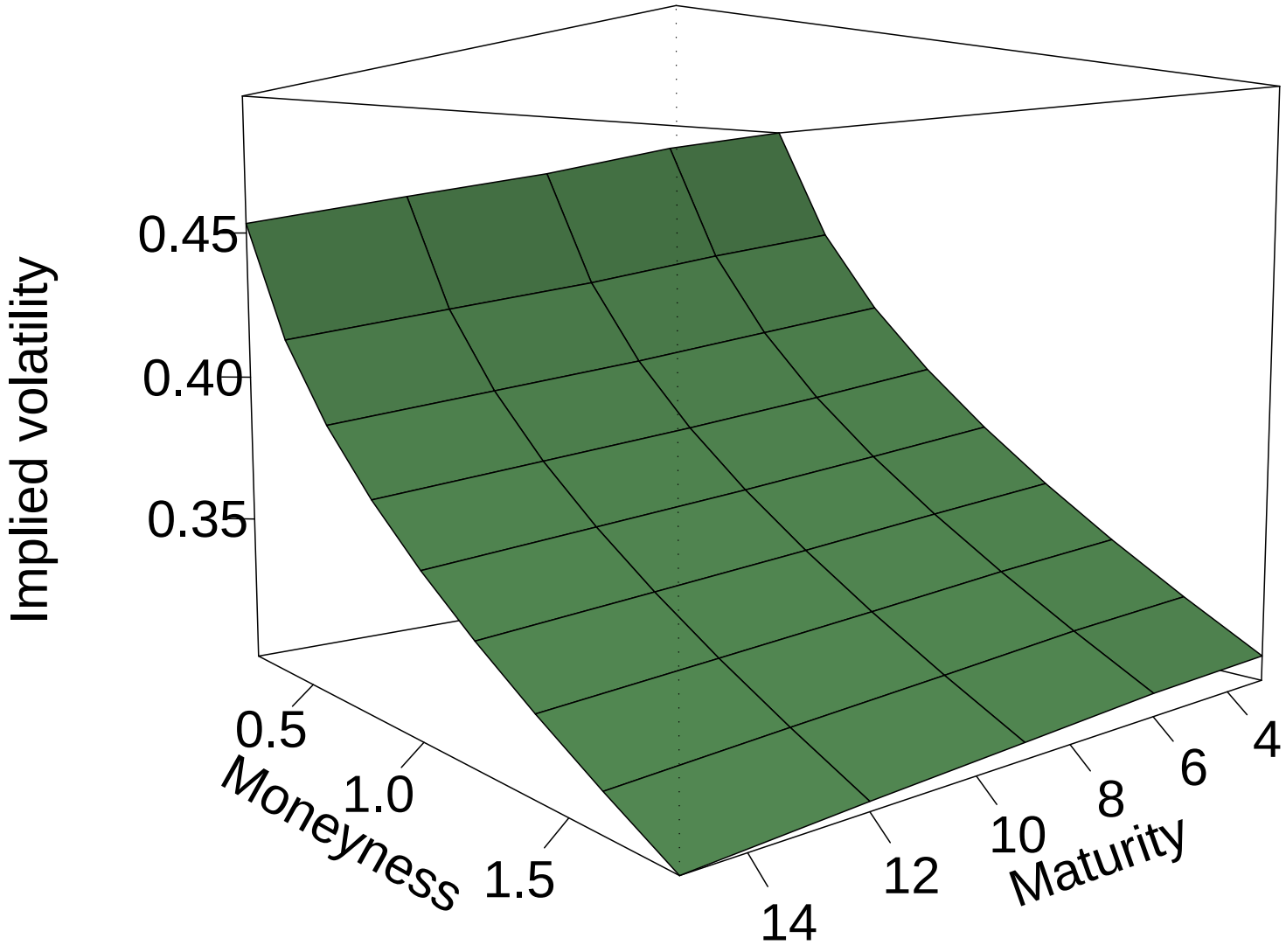
Risk and Return



Risk and Return

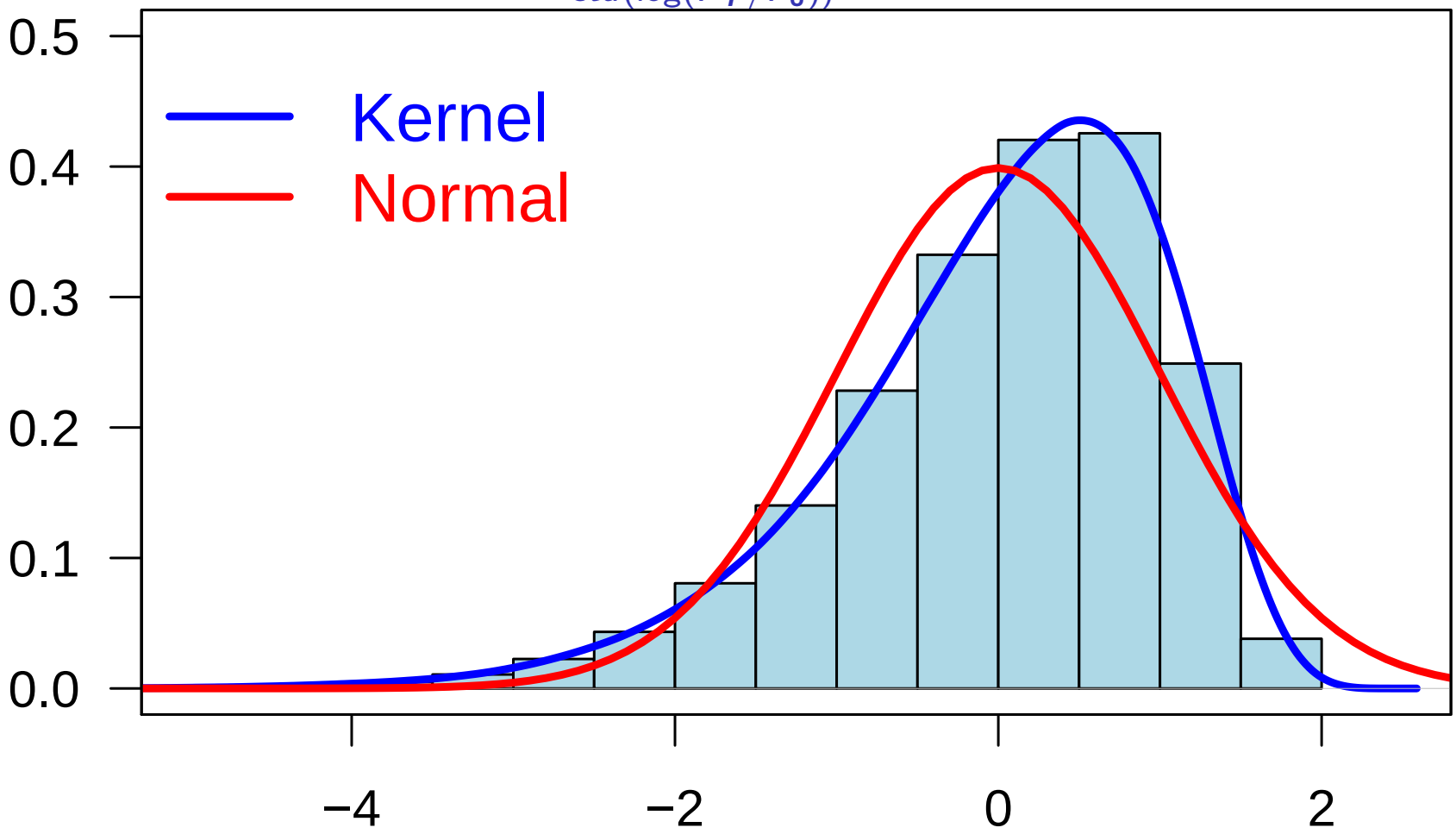


Implied Volatility and Time Horizon

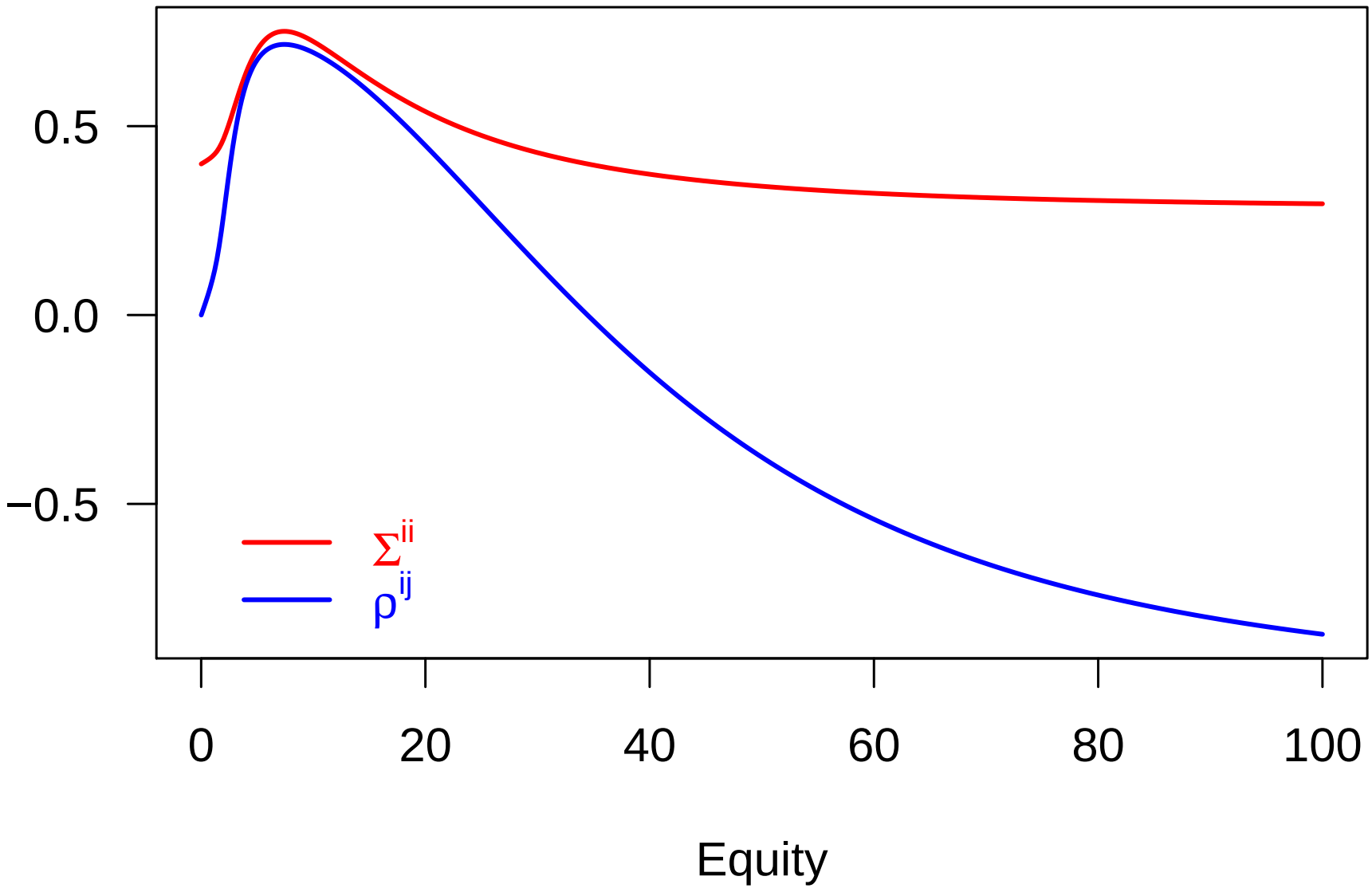


Payoff Density

$$\frac{\log(P_T)-\log(mean(P_T)))}{std(\log(P_T/P_0))}$$



Endogenous Correlation



Paper with EWMA

- Before, REE was used to get the mean and covariance forecasts
- Here we want to use more heuristic methods — for now EWMA
- Has many advantages because of its flexibility

Questions and Research direction

- Contagion
  - From one asset to another (e.g. correlations)
  - From one institution to another
- Minsky moments — history–dependence
  - Long period of stability creates conditions for instability
- Under what market conditions does speculative trading increase/decrease market stability
- Is harmonization (and higher quality) of risk management systems beneficial — competing risk systems
- Estimation

its still early days for this research dirrection

Setup

maximization problem

$$\max_{D_t} \; rV_t + D_t^\top(\mu_t - r) \qquad \text{subject to} \qquad \alpha\sqrt{D_t^\top \Sigma_t^\top D_t} \leq V_t$$

evolution of wealth

$$dV_t = V_{t+1} - V_t = \left[rV_t + D_t^\top(\mu_t - r)\right] + D_t^\top \sigma_t dW_t$$

budget constraint

$$V_t = b_{t-1}B_t + \sum_i \theta_{t-1}^i P_t^i$$

Updating of beliefs

- At the moment we just do EWMA

$$\mu_t = \lambda^{\mu}\mu_{t-1} + (1-\lambda^{\mu})R_t$$
$$\Sigma_t = \lambda\Sigma_{t-1} + (1-\lambda)R_t'R_t$$

- but this could be replaced with any equation.
- Allows us to explore alternative risk management systems
- and competition among risk management system

Solve simultaneously

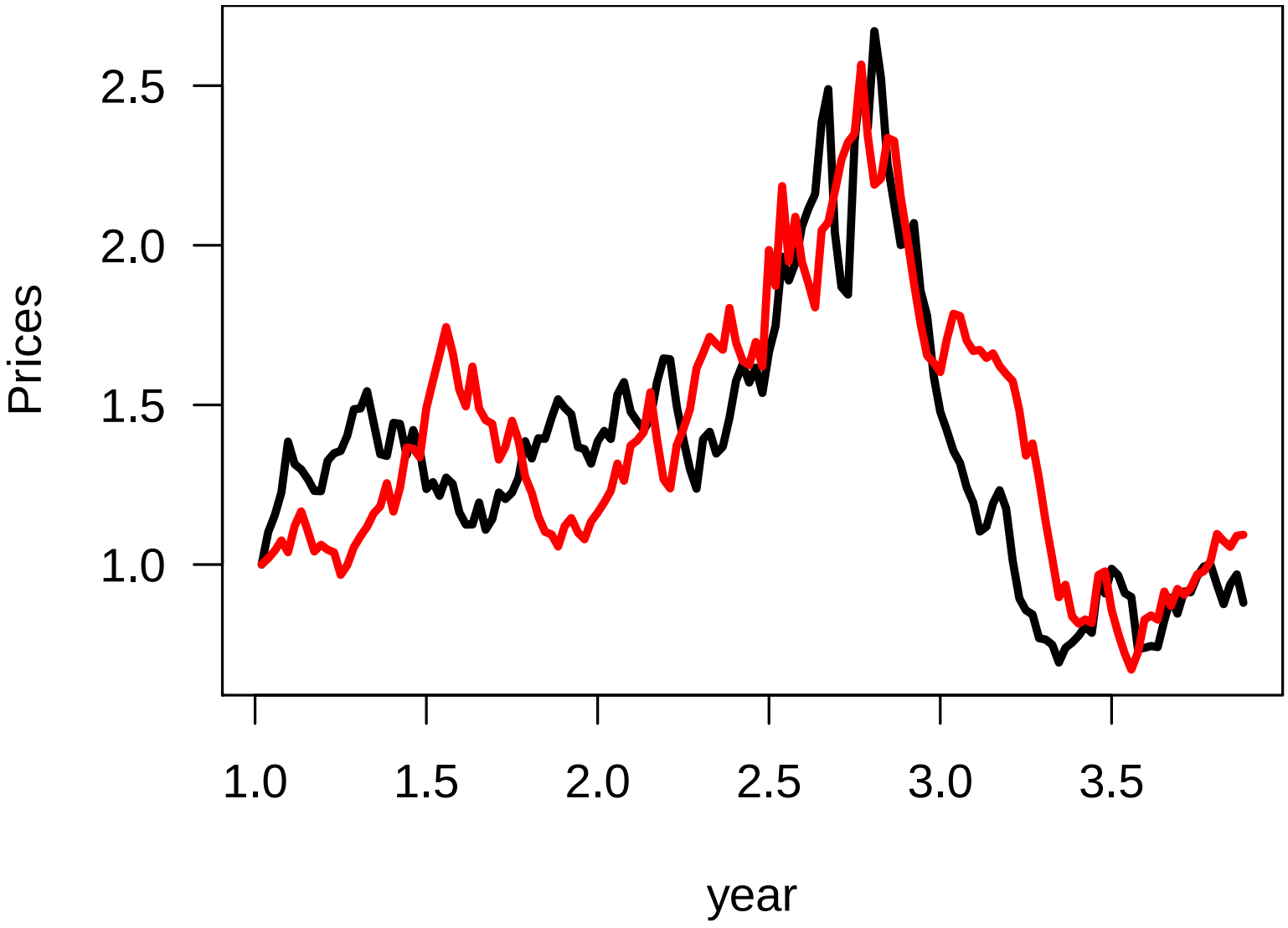
$$
\begin{aligned}
R_t^i &= \log\left(\frac{P_t^i}{P_{t-1}^i}\right) \\
\mu_t &= \lambda^\mu \mu_{t-1} + (1-\lambda^\mu) R_t \\
\Sigma_t &= \lambda \Sigma_{t-1} + (1-\lambda) R_t' R_t \\
\xi_t &= (\mu_t - r)^\top \Sigma_t^{-1} (\mu_t - r) \\
P_t^i &= \exp\left(\frac{V_t}{\alpha \delta^i \sqrt{\xi_t}}(\mu_t^i - r) + rt + \eta z_t^i\right) \\
V_t &= b_{t-1} B_t + \sum_i \theta_{t-1}^i P_t^i
\end{aligned}
$$

The price impact function

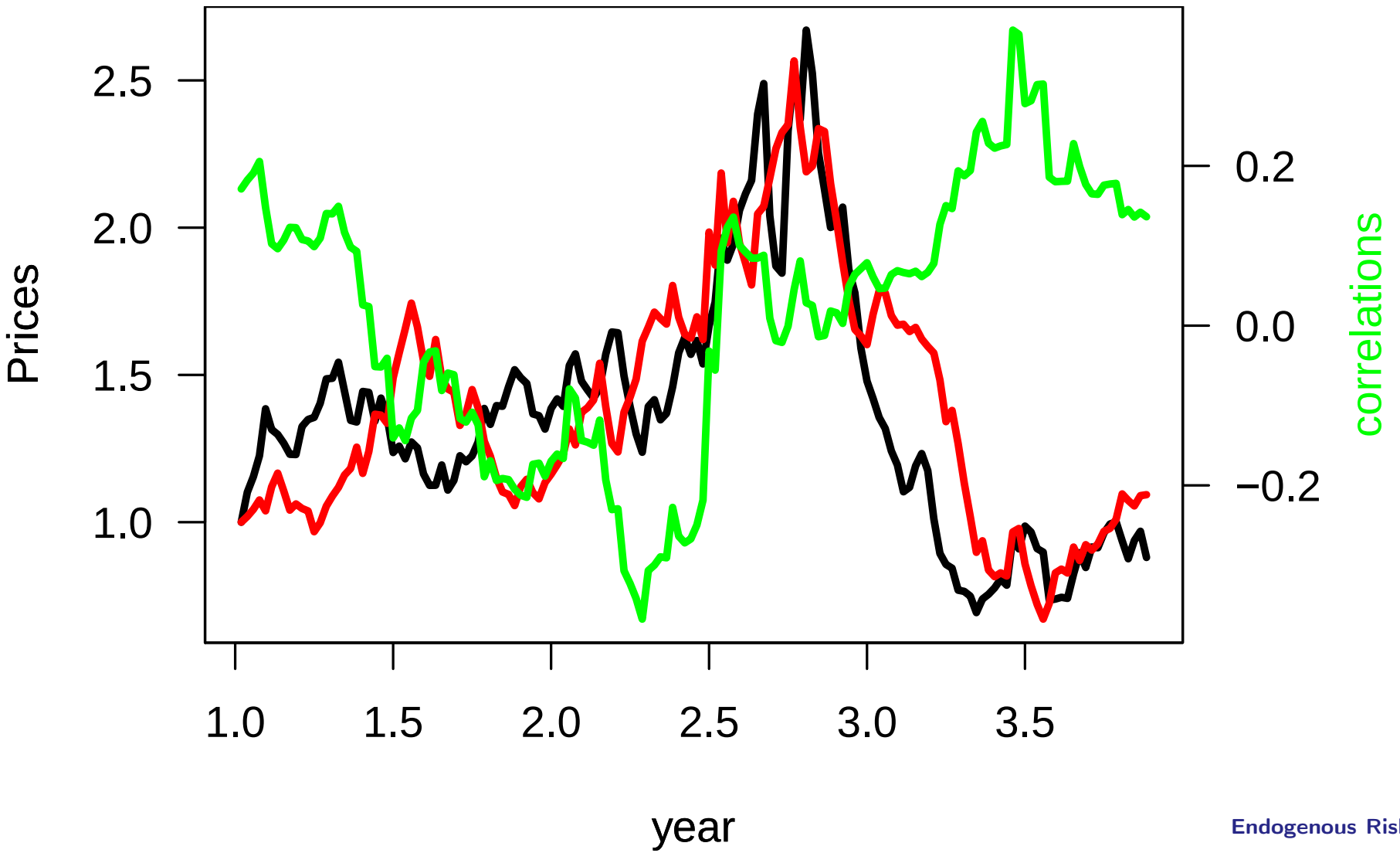
$$P_t^i = \exp\left(\frac{V_t}{\alpha\delta^i\sqrt{\xi_t}}(\mu_t^i - r) + rt + \eta z_t^i\right)$$

- z exogenous shocks (IID normal)
- α, δ the price impact parameters
- But is this the best function?
- It has some problems

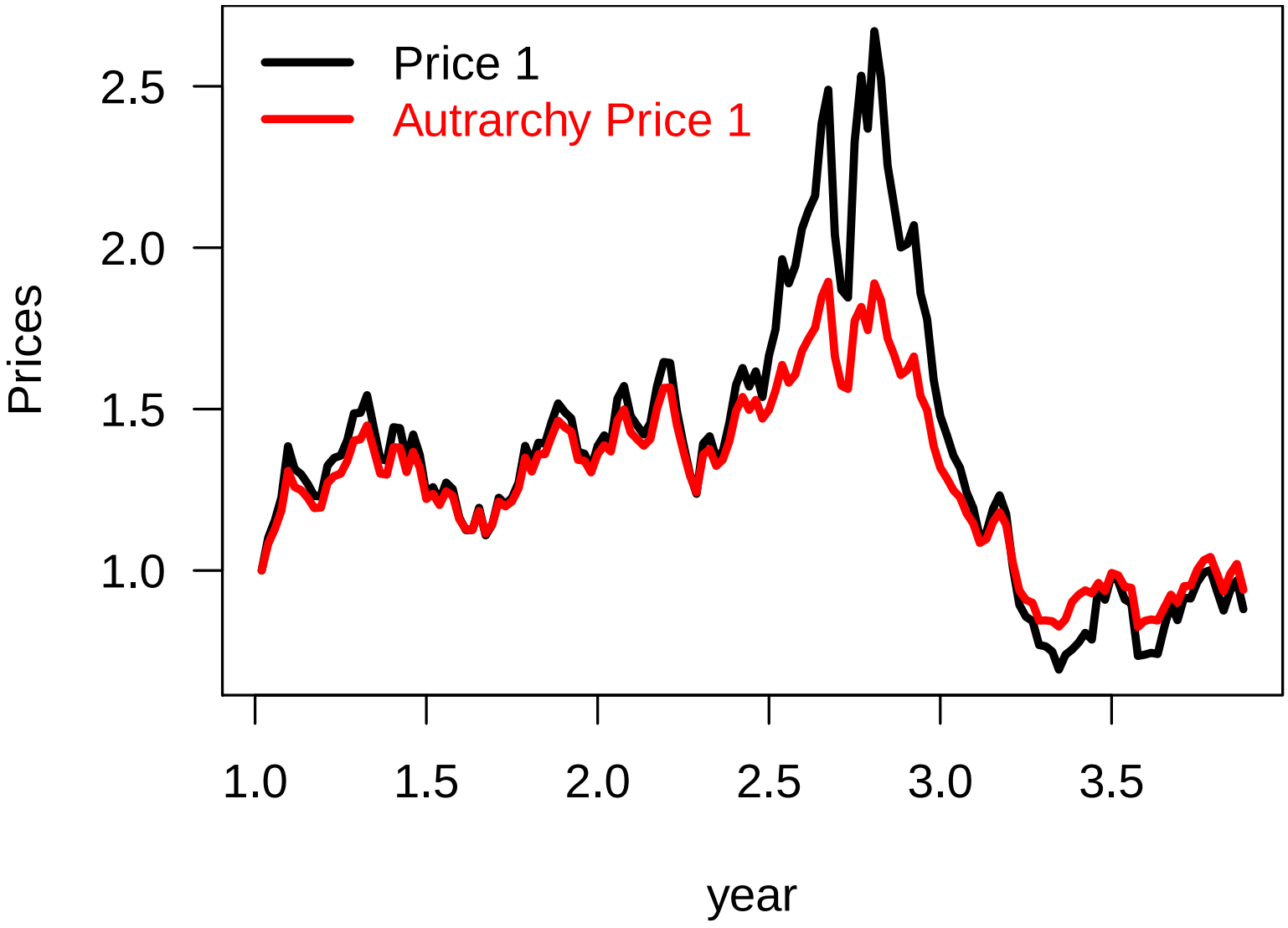
Prices of 2 assets



Prices of 2 assets and their EWMA correlation



Price of Asset 1 and Autarchy Price



Related Literature

- Two strands coming together
  - Competitive equilibrium models of crashes: Leland (1990), Genakoplos (1997) and Geanakoplos and Zame (2003)
  - Corporate finance elements: Shleifer and Vishny's (1997), Holmström and Tirole (2001), He and Krishnamurthy (2007)...
- Portfolio constraints: Basak and Croitoru (2000), Chabakauri (2008), Aiyagari and Gertler (1999), Gromb and Vayanos (2002), Brunnermeier and Pedersen (2007), Rytchkov (2008), Pavlova et al (2008)

- Wealth effects: Kyle and Xiong (2001). Xiong (2001) solves for fixed point numerically.
- Lagrange multipliers associated with VaR constraints: Danielsson, Shin and Zigrand (2004), Brunnermeier and Pedersen (2008), Danielsson and Zigrand (2008), Oehmke (2008).
- Asset pricing taking account of balance sheet constraints: Adrian, Etula and Shin (2009) [for exchange rates], Etula (2009) [commodities], Adrian, Moench and Shin (2009)